1. (20) Evaluate each of the following sets, where

A = {2, 8, 12, 16}

B = {4, 0, 8, 16}

C = {12, 4, 16, 14}

U = {0, 2, 4, 6, 8, 10 ,12, 14, 16}

a) $A \cap B \cap C =$

b) $B - (A - C) =$

c) $A' \cap B' =$

d) $(C')' =$

e) $A \cup B \cup C =$

Is it a partition of U?

2. (10) Prove the following statement.

If $A \subseteq B$ and $B \subseteq A$, then $A = B$

3. (10) Describe the following diagrams using sets A, B, A′, B′, U and operations ∪ (union), ∩ (intersection), and – (complement)

4. (20) For the given f: A → B, determine whether f is surjective, injective, or bijective. Justify your answer.

a) A = Ì x Ì , B = Ì , $f(x,y) = x^2 + y^2$

b) A = {0, 1} x Ì+, B = {0,1} , $f(x,y) = x^y$

c) A = Ì , B = Ì , $f(x) = 2x + 5$

d) A = Ì , B = Ì , $f(x) = \begin{cases} 0 & \text{if } x \text{ is even} \\ \frac{x-3}{2} & \text{if } x \text{ is odd} \end{cases}$

5. (10) Let S be a set of four elements { 0, 1, –1 , 2 } .

In the table below the result x * y (binary operation) is found in the row that starts with x at the left and in the column that has y at the the top. For example, 1 * (–1) = 1

*	0	1	–1	2
0	1	–1	0	1
1	2	1	1	–1
–1	0	1	–1	2
2	1	–1	2	0

a) Find an identity element in S for *.

b) List all elements which have right inverses.

c) List all elements which have left inverses.

d) List all elements which have two-sided inverses.

e) Is the binary operation * commutative?

6. (10) In each part below, a relation R is defined on the set Ì. Determine whether R is reflexive, symmetric, or transitive.

a) x R y if and only if $y = 2^x$

b) x R y if and only if $x - y = 4$

7. (10) Show that the following statements are true using Principle of Mathematical Induction.

a) $1 + 2 + 2^2 + 2^3 + . . . + 2^n = \frac{1-2^{n+1}}{1 - 2}$

b) If $x \in$ Ì and $x \neq 0$, then $x^{4n} \in$ Ì+,
for every $n \in$ Ì+

8. (10) For a and b as given below, find q and r such that

$a = b \cdot q + r, \quad o \leq r < b$

1) $a = 135$, $b = 268$

2) $a = -18$, $b = 3$