MATH 222 NAME_____

DATE MARCH 27, 1991

ALGEBRAIC STRUCTURES

MIDTERM #2

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1. (15) In each part, find the greatest common divisor (a,b) and integers m and n such that (a,b) = am + bn.

a) (10) $a = 5 \ , \ b = -12$

b) (5) $a = 63 \ , \ b = 72$

2. (15) Use Euclidean Algorithm to find a solution $x \in \mathbb{Z}$, $0 \leq x \leq n$, for each of the congruences $ax \equiv b \pmod{n}$. Note that in each case a and n are relatively prime.

a) (10) $15x = 24 \pmod{31}$

b) (5) $5x = 25 \pmod{62}$

3. (20) Write down a multiplication table for $\mathbb{Z}_5$ and list all elements of $\mathbb{Z}_5$ that have multiplicative inverses.

4. (10) Determine if the following sets G with the operation indicated form a group. If not, point out which of the group axioms fail.

a) G = set of all integers, $a * b = a - b$

b) G = set of all non-negative integers, $a * b = a + b$

Note: $0 \in G$.

5. (15) Let $G = \langle a \rangle$ be a cyclic group of order 15.

a) List all the distinct subgroups of G

b) List all the distinct generators of G

c) Suppose $G = \mathbb{Z}_{15} = \langle [2] \rangle$ under addition.
List all the distinct generators of $\mathbb{Z}_{15}$.

6. (15) Consider two groups G_1 and G_2 defined by the following tables:

$G_1 = \{1, -1\}$ $\qquad\qquad$ $G_2 = \mathbb{Z}_2$

*	1	−1
1	1	−1
−1	−1	−1

+	[0]	[1]
[0]	[0]	[1]
[1]	[1]	[0]

Let $\Phi : G_1 \to G_2$ be defined by

$\Phi(1) = [0]$

$\Phi(-1) = [1]$

a) (10) Is Φ an isomorphism?

b) (5) Are G_1 and G_2 isomorphic to each other?

7. (10) a) Compute $g\ f\ g^{-1}$ for the pair

$f = (2, 4, 6)\ (3\ 5\ 7)$

$g = (1, 2, 4)\ (3\ 6\ 7)$

b) For the given permutations f and h , find a permutation g such that $g\ f\ g_{-1} = h$

$f = (2\ 3)\ (5\ 6\ 7)$

$h = (1\ 5)\ (4\ 6\ 7)$