MATH 222 NAME____________________

DATE APRIL 15, 1991

ALGEBRAIC STRUCTURES

MIDTERM #3

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1. (15) Determine in each of the parts if the given mapping $\Phi: G \to G'$ is a homorphism. If so, identify its kernel. Justify your answer.

(In what follows G and G' are groups).

a) $G = \mathbb{Z}$ under $+$, $G' = \mathbb{Z}_n$ under $+$

$\Phi(a) = [a]$ for $a \in \mathbb{Z}$

b) $G = R^+$ (all positive reals) under multiplication,

$G' = R$ under addition,

$\Phi(a) = (\log_7 a)$ for $(a \in R^+)$

c) G abelian group,

$\Phi: G \to G$ defined by

$\Phi(a) = a^{-1}$ for $a \in G$

2. (10) Show that if $\Phi : G \rightarrow G'$ is a homomorphism from group G to group G', then :

(a) $\Phi(e) = e'$, the unit element of G'

(b) $\Phi(b^{-1}) = \Phi(b)^{-1}$ for all $b \in G$

3. (15) Decide if each of the following sets is an integral domain. Is it a field? Justify your answers.

(a) $\mathbb{Z}_8$ under usual addition and multiplication.

(b) $\mathbb{Z}_{23}$ under usual addition and multiplication

(c) The set of all real numbers of the form

$$m + n\sqrt{5} \quad , \text{ here } \quad m, n \in \mathbb{Z} \, ,$$

with the usual addition and multiplication of real numbers.

4. (20) For every group G and subgroup H find index of H in G.
Is H a normal subgroup? Justify your answers.

(a) (5) $G = S_3$, symmetric group, $H = \{ (1) \}$

(b) (15) $G = S_3$, $H = \{ (1), (1,2) \}$

5. (20) Let G be the group of all non zero real numbers under multiplication and N be the subgroup of all positive real numbers.

(a) (10) Write out G/N by exhibiting the cosets of N in G.

(b) (10) Construct multiplication table for G/N and show that G/N is a group.

6. (20) Let R be the ring of all 2 x 2 matrices over Î (with respect to addition and multiplication of matrices). Consider the subset S of R that consists of all 2 x 2 matrices of the form.

$$\begin{pmatrix} a & b \\ 0 & a \end{pmatrix} \quad , \text{ where } a, b \in \text{Î}.$$

(a) (10) Show that S is a subring of R.

(b) (10) Is it a field? Justify your answer.