

Name: _____

Instructor: Heide Gluesing-Luerssen

Mathematics 222.02: Algebraic Structures
Spring Semester 1998
Exam 2
March 6, 1998

This Examination contains 6 problems on 5 sheets of paper including the front cover. Do all your work in this booklet and show your computations. Calculators, books, and notes are not allowed.

Scores

Question	Possible	Actual
1	15	
2	15	
3	15	
4	20	
5	20	
6	15	
Total	100	

Sign the pledge:

“On my honor, I have neither given nor received unauthorized aid on this Exam.”

Signature: _____

GOOD LUCK

1. (a) Find all units in \mathbb{Z}_{21} .

(b) Calculate the quotient $\frac{15}{10}$ in \mathbb{Z}_{21} , that is, find $15 \cdot 10^{-1}$.

2. Find all greatest common divisors of $11 - 10i$ and $-13i$ in $\mathbb{Z}[i]$ and give a Bezout-equation for one of them.

3. Let a and b be relatively prime numbers. Assume that there are numbers x and y so that

$$ax \equiv ay \pmod{b}.$$

Show that it follows $x \equiv y \pmod{b}$.

4. (a) Show that the polynomial $p(X) = 2X^2 + 3X - 7$ has no roots in \mathbb{Z} .

(b) Show that $\sqrt[25]{2}$ is irrational.

5. Consider the ring \mathbb{Z}_{41} .

(a) Assume that $a \neq 1$ is an element of \mathbb{Z}_{41} with $a^{15} = 1$. Show that the order of a is 5.

(b) Find the order of 2. [Make sure that you get a divisor of 40.]

(c) Find an element of order 5 in \mathbb{Z}_{41} .

(d) Use the list you produced in (b) to find a square root of -1 in \mathbb{Z}_{41} .

(e) How many square roots of 2 does \mathbb{Z}_{41} have? (You don't have to find any!)

(f) Calculate the fraction $\frac{32}{10}$ in \mathbb{Z}_{41} , i. e. find $32 \cdot 10^{-1}$.

6. (a) Find a Bezout-equation for $a = 8$ and $b = 35$.

(b) Use (a) to find a solution of the equation $8X \equiv 7 \pmod{35}$.