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Mathematics 222.02: Algebraic Structures
Spring Semester 1998
Exam 3
April 20, 1998

This Examination contains 8 problems on 5 sheets of paper including the front cover. Do all your work in this booklet and show your computations. Calculators, books, and notes are not allowed.

Question	Possible	Actual
1	10	
2	15	
3	10	
4	15	
5	10	
6	15	
7	10	
8	15	
Total	100	

Sign the pledge:

"On my honor, I have neither given nor received unauthorized aid on this Exam."

Signature: ______________________________

GOOD LUCK

1. Give three different ways to write the integer 252 as a product of prime numbers.

2. Show that there are infinitely many prime numbers.

3. In $\mathbb{Z}[i]$ the following equation is true

$$5 = (2+i)(2-i) = (1+2i)(1-2i)$$

(a) Show that both products are factorizations of 5 into Gaussian primes.

(b) How does the result of (a) relate to the fact that $\mathbb{Z}[i]$ is a unique factorization domain? Give the exact relationship between the two factorizations.

4. Factorize the following numbers into primes in $\mathbb{Z}[\sqrt{2}]$.

(a) $4 - 5\sqrt{2}$,

(b) 17,

(c) $87 - 58\sqrt{2}$,

(d) 83.

5. An n-cycle is a cycle of length n, i. e. a permutation of the form $(a_1\ a_2\ \ldots a_n)$ with different a_i.
Prove: A_n contains an n-cycle exactly when n is odd.

6. Consider the symmetric group S_6.

(a) Give three different elements of order 6 in S_6.

(b) Give a subgroup of S_6 with order 24. (You don't have to list all elements of this subgroup!).

(c) Give a subgroup of S_6 with order 5.

7. Let $\sigma = (1\,5)(2\,4\,7\,3)$ and $\tau = (3\,5\,6\,2)(1\,4\,7)$ in S_7.

(a) Calculate $\operatorname{sgn} \sigma$.

(b) Calculate $\sigma\tau^2\sigma^{-1}$.

(c) Find a permutation $\rho \in S_7$ so that $\rho\sigma = (2\,5\,1\,4\,7\,3)$.

8. **Prove by induction:** $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \ldots + (n-1) \cdot n = \dfrac{(n-1)n(n+1)}{3}$ for all $n \geq 2$.

Give all details of the induction.