Chris Bendel and Peter Cholak Math 222 - Exam 1 Wednesday, February 24

Be sure to carefully write up your answers. Be sure to explain your answers.

(4 points each) **Define** the following terms:

a) $\sqrt[n]{1}$ (over the complex numbers for a positive integer n).

b) A *constructible* number.

c) A *primitive* nth root of unity for a positive integer n.

d) The *multiplicative inverse* of a number a in $\mathbb{Z}_n$ (n a positive integer).

(2 points each) Answer **True** or **False** - no work required:

a) If $\arg(z) = \phi$, then $\arg(z^2) = \phi^2$.

Answer: ____________________

b) $|3 + 4i| = 5$.

Answer: ____________________

c) $5 \cdot 6 \equiv 5 \pmod 7$.

Answer: ____________________

d) The coefficient of x^8y^2 in $(x + y)^{10}$ is 45.

Answer: ______________________

(15 points) Find all solutions to $x^6 - 8 = 0$. Explain briefly why these solutions are constructible? (You do not need to simplify your answer to the form $a + bi$.)

(15 points) Find the multiplicative inverse of 56 in $\mathbb{Z}_{67}$.

(15 points) Prove that $n^5 - n$ is divisible by 30 for any positive integer n.

(15 points) Let n be a positive integer and ζ be the first nth root of unity (i.e. $\zeta \neq 1$) and let $\alpha = \zeta^k$ be any other nth root of unity. Prove that there exists an integer m such that $\zeta = \alpha^m$ if and only if k has a multiplicative inverse in $\mathbb{Z}_n$.

(15 points) Let a and b be nonzero integers such that $g = (a, b)$. Prove that $\left(\frac{a}{g}, \frac{b}{g}\right) = 1$.