Chris Bendel and Peter Cholak Math 222 - Sample Final Thursday, May 6

The final exam will be Thursday, May 6 from 1:45 pm to 3:45 pm in 118 DBRT (Prof. Cholak's section) or Nieuwland 127 (Prof. Bendel's section).

The final exam will cover all the material discussed in class through Chapter 9. Fair game for the final are all questions (or similar questions) on the previous exams and sample exams, homework, and quizzes. The best way to prepare for the final is to be sure you can do all problems on this sample final, the previous two sample exams, previous two exams, and the quizzes.

The exam will consist of four parts. Part I: Definitions – There will be six terms to define, worth 4 points each (24 points in all). Part II: True/False – There will be six true/false questions, worth 2 points each (12 points in total). Part III: Computational Problems – there will be some number of computational problems for a total of 70 points. Part IV Proofs – There will be three proofs, worth 15 points each (or 45 points total). There is one bonus point. The following are some examples from the latest material along some other possible problems.

Part I: Definitions.

(4 points each) **Define** the following terms:
a) *Isomorphism of groups.*
b) *Coset.*
Part II: True/False.

(2 points each) Answer **True** or **False** - no work required:
a) The quaternions are isomorphic to D_4.
b) The quaternions are cyclic.
Part III: Computational Problems.

List all elements of $\sqrt[12]{1}$ and determine the orders of all elements.

Find all the primitive 16th roots of unity in $\mathbb{Z}_{17}$.

Consider the map $f : (\mathbb{Z}_8, +) \to (\mathbb{Z}_8, +)$ by $f(x) = 4x$. Is this an isomorphism of groups? Why or why not?

Identify all the subgroups of S_3.

Find all the possible orders of subgroups of D_5. Give examples of subgroups of each of these orders. Compute the index of each of these subgroups in D_5.

Let $G = \left\{ \begin{pmatrix} a & a \\ a & a \end{pmatrix} : a \in \mathbb{R}^* \right\}$. G is a group under matrix multiplication.
a) Find the identity element of this group.
b) Find the inverse of $\begin{pmatrix} 3 & 3 \\ 3 & 3 \end{pmatrix}$ in G.

The set $G = \{5, 15, 25, 35\}$ is a group under multiplication modulo 40.
a) Find the identity element in G.
b) Find the inverses of the remaining elements in G.
c) Which group of order 4 is G isomorphic to? Why?

Part IV: Proofs. There will be three proofs, worth 15 points each. These problems will come from either of the first two sample exams or the following:

For each positive integer n, show that a group of order n has exactly 2^{n-1} *subsets* which contain 1_G. Hint: Use induction.

Let $f : G \to H$ be an isomorphism of groups. Prove that $f(1_G) = 1_H$.

Let p be a prime number and G be a finite group of order p.

a) Show that G is cyclic.

b) Use part (a) to show that there is only one group of order p up to isomorphism.

Show if G has order p^n (for $n \geq 1$) then G has an element whose order is p.

Let G be a finite group and $H \subset G$ be an nonempty subset. Suppose that H is closed under multiplication. Show that H is in fact a subgroup of G.

Let h be a function from a group $(G, *)$ to a group $(H, *)$ such that $h(a * b) = h(a) * h(b)$. The kernel of h is $\{g \in G : h(g) = 1_H\}$. Show the kernel of h is a subgroup of G.