Chris Bendel and Peter Cholak Math 222 - Final Thursday, May 6

(4 points each) **Define** the following terms:

a) A *subgroup*.

b) The *modulus* of a complex number z.

c) A polynomial being *solvable by radicals*.

d) An *odd permutation*.

e) A *cyclic* group.

(2 points each) Answer **True** or **False** - no work required:

a) Every degree 3 polynomial over $\mathbb{Q}$ has a nonconstructible zero.

b) There is a degree 5 polynomial over $\mathbb{Q}$ which is not solvable by radicals.

c) Let H be a subgroup of the group G and let h be an element of H. Then the group $< h >$ is contained in H.

d) The order of (1 2 3) is even in S_3.

e) The quaternions are isomorphic to a subgroup of $(\mathbb{Z}_{16}, +)$.

f) Let $f : (\mathbb{C}, +) \to (\mathbb{C}, +)$ by $f(a + bi) = a - bi$. f is an isomorphism.

(10 points) Find all the zeros of $x^5 + 32$ in the complex numbers.

(10 points) Find all the primitive 6th roots of unity in $\mathbb{Z}_7$.

(10 points) Does S_8 have a cyclic subgroup of order $3, 5, 7, 11, 13, 15$? If so find such a subgroup.

(10 points) Are D_6 and A_4 isomorphic groups? Why or why not?

(10 points) Use the Euclidean algorithm to find $d = \gcd(304, 399)$ and then find integers x and y such that $d = 304x + 399y$.

(10 points) Construct the cyclic table for $GF(3, x^2 + 2x + 2)$.

(10 points) Given that $3i$ is a zero of $f(x) = x^3 - 6ix^2 - 11x + 6i$ over $\mathbb{C}$. Find all the zeros of $f(x)$ in $\mathbb{C}$.

(15 points) Let p be a prime number and $P(x)$ be an irreducible polynomial of degree ν over $\mathbb{Z}_p$. Suppose that n is a positive integer relatively prime to $p^\nu - 1$. Prove that there is exactly one nth root of unity in $GF(p, P(x))$.

(15 points) Let f be a function from a group $(G, *)$ to a group $(H, *)$ such that $f(a * b) = f(a) * f(b)$. The kernel of f is $\{g \in G : f(g) = 1_H\}$. Show the kernel of f is a subgroup of G.

(15 points) Let p be a prime number and G be a finite group of order p.

a) Show that G is cyclic.

b) Use part (a) to show that there is only one group of order p up to isomorphism.