Chris Bendel and Peter Cholak Math 222 Wednesday January 20

Be sure to carefully write up your answers. It is suggested that you first write out a draft of your proposed questions and then carefully rewrite that draft to get your final version. You do *not* have to write the answers on this sheet of paper.

In the "real" world, we are used to the fact that a product of two non-zero numbers is again non-zero. We will later see that this fact is not true in all worlds. Show that the complex numbers do behave normally. Precisely, Let z and w be two arbitrary complex numbers and suppose that the product $zw = 0$. Show that at least one of z or w must be zero.

Show $(1)(2)(3)+(2)(3)(4)+(3)(4)(5)+\ldots+n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}$.

The Fibonacci numbers are 1, 1, 2, 3, 5, 8, 13, 21, 34, ... In general, the Fibonacci numbers are defined by $f_1 = 1$, $f_2 = 1$, and $f_{n+2} = f_{n+1} + f_n$ for $n = 1, 2, 3, \ldots$. Prove that the nth Fibonacci number f_n satisfies $f_n < 2^n$. **Hint:** Use the "second" version of induction and you will have to prove *two* starting cases.