

Chris Bendel and Peter Cholak Math 222 Wednesday, February 10

Be sure to carefully write up your answers. It is suggested that you first write out a draft of your proposed questions and then carefully rewrite that draft to get your final version. You do *not* have to write the answers on this sheet of paper.

Let  $p$  be a prime number. Show that every  $p$ th root of unity, except 1, is primitive.

Consider the set of integers modulo 8,  $\mathbb{Z}_8$ .

- (a) Write out the addition and multiplication tables for  $\mathbb{Z}_8$ .
- (b) Identify the additive inverse of each element in  $\mathbb{Z}_8$ .
- (c) Identify those elements which have a multiplicative inverse and what their inverses are.
- (d) What is the sum  $1 + 2 + 3 + 4 + 5 + 6 + 7$  modulo 8?

Let  $n$  be a positive integer and consider the sum  $1 + 2 + 3 + \cdots + n - 1$  modulo  $n$ . Show that if  $n$  is odd, this sum is zero in  $\mathbb{Z}_n$  and if  $n$  is even then the sum is  $n/2$  in  $\mathbb{Z}_n$ . *Hint:* Use the formula for this sum which we saw when discussing induction.