Chris Bendel and Peter Cholak Math 222 - Quiz 7 Wednesday, March 17 Be sure to carefully write up your answers. It is suggested that you first write out a draft of your proposed questions and then carefully rewrite that draft to get your final version. You do *not* have to write the answers on this sheet of paper.

Problem 4 from Section 6.3.

Consider the Galois Field  $F \equiv \mathrm{GF}(3, x^2 + x + 2)$ . Let  $\alpha$  be the associated Galois imaginary.

- (a) Compute all powers of  $\alpha$ , i.e.  $\alpha$ ,  $\alpha^2$ ,  $\alpha^3$ , .... What is the least power for which  $\alpha^k = 1$ ?
  - (b) Find the inverse of each nonzero element in  $F.\ Hint:$  Use part (a).
- (c) By definition  $\alpha$  is one solution to  $x^2 + x + 2 = 0$  over  $\mathbb{Z}_3$ . There should of course be another solution. It is also an element of F. Find this element. Is it a power of  $\alpha$ ? *Hint*: Use long division.