Peter Cholak and Juan Migliore Math 222 - Sample Exam 1 Friday, February 23

The exam will cover everything we have covered in class except the general solutions of a cubic, and will have format as outlined below.

(5 parts at 4 points each) **Define** the following terms:
a) $\sqrt[6]{4}$ (over the complex numbers).
b) The *argument* of a complex number z.
c) The *order* of an nth root of unity for a positive integer n.
d) The *additive inverse* of a number a in $\mathbb{Z}_n$ (n a positive integer).
e) *Constructible.*

(5 parts at 2 points each) Answer **True** or **False** - no work required:
a) $\arg(1+i) = \pi/4 + 2\pi n$ for any $n \in \mathbb{Z}$.
b) For any $z \in \mathbb{C}$, $|z|^3 = 3|z|$.
c) $3 \cdot 17 \equiv 3 \pmod 6$.
d) The coefficient of a^7b^4 in $(a+b)^{11}$ is 330.
e) the possible orders of elements in $\mathbb{Z}_7$ are $1, 2, 3, 4, 6$.

Part III: Computational Problems - there will be four problems something *like* the following for a total of 40 points.

Find all solutions to $x^5 - 32 = 0$. Are these solutions constructible? Why or why not? (You do not need to simplify your answer to the form $a + bi$.)

Find the multiplicative inverse of 14 in $\mathbb{Z}_{59}$.

Prove that $n^{13} - n$ is divisible by 78 for any positive integer n.

Does $266x + 361y = 57$ have integer solutions? If so find them. It not, why not? (You must use the Euclidean algorithm as discussed in class.)

Find all roots of $x^{4666666} + x^2 + 3$ in $\mathbb{Z}_5$. Find all the primitive elements in $\mathbb{Z}_{19}$

Part IV: Proofs - *exactly* two of the following problems will appear on the exam. They'll be worth 15 points each. Remember that "$a \equiv_n b$" is another way of writing "$a \equiv b \pmod n$".

Show $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \ldots + n(n+1) = \frac{n(n+1)(n+2)}{3}$.

The *least common multiple* of x and y is the least positive number which is a multiple of both x and y. Prove that the least common multiple of any two positive integers k and m is $\dfrac{km}{(k,m)}$.

Let a and b be nonzero integers such that $g = (a, b)$. Prove that $\left(\dfrac{a}{g}, \dfrac{b}{g}\right) = 1$.

For any prime p, if $a^p \equiv_p b^p$ then $a^p \equiv_{p^2} b^p$. (Hint: use Proposition 5.3.)

Let m, n and k be integers such that $(m, n) = 1$ and both m and n divide k. Then prove that the product mn divides k.

Let p be prime. Show every imaginary (complex and not real) pth root of unity is a primitive pth root of unity.