Peter Cholak and Juan Migliore Math 222 - Exam 1 Wednesday, February 28

Except where noted, be sure to show all your work.

(4 points each for a total of 20 points) **Define** the following terms:

a) $\sqrt[n]{z}$ over the complex numbers.

b) A *primitive* complex nth root of unity.

c) m and n are *relatively prime.*

d) The *multiplicative inverse* of a nonzero number a in $\mathbb{Z}_p$ (p a prime).

e) An equation $a_0x^n + a_1x^{n-1} + \dots a_{n-1}x + a_n = 0$ is *solvable by radicals* (or *algebraically resolvable*) if ...

(2 points each for a total of 10 points) Answer **True** or **False** - no work required:

a) the possible orders of elements in $\mathbb{Z}_{19}$ are 1 and 19.

b) 4 is the multiplicative inverse of 3 in $\mathbb{Z}_{11}$.

c) The coefficient of a^6b^8 in $(a^2 - 2b)^{11}$ is divisible by 11.

d) 3 has a multiplicative inverse in $\mathbb{Z}_{18}$.

e) 3 is an primitive element of $\mathbb{Z}_{13}$.

(10 points) Find all complex solutions to $x^6 - 4x^3 + 3 = 0$. Which of these solutions are constructible? Why or why not? (You may use the first third root of unity, ω, in your answer.)

(10 points) Find the multiplicative inverse of 23 in $\mathbb{Z}_{31}$.

(10 points) Show that for all odd k, $n^k - n$ is divisible by 3.

(10 points) It happens to be true that 1997 and 1999 are both prime numbers (you don't have to check this). Explain why the polynomial $x^{1997} - 1$ has no roots in $\mathbb{Z}_{1999}$ other than $x = 1$. (Hint think about the order of the root.)

(15 points) Let a and b be nonzero integers such that $g = (a, b)$. Prove that $\left(\frac{a}{g}, \frac{b}{g}\right) = 1$.

(15 points) For any prime p, if $a^p \equiv_p b^p$ then $a^p \equiv_{p^2} b^p$. (Hint: use Proposition 5.3.)