Peter Cholak and Juan Migliore Math 222 - Sample Exam 2 Friday, April 20

The exam will *focus* on sections 6.1-6.3, 7.1-7.3, 8.2, 8.4, and 9.1-9.2 (the material we covered since the first exam) and will have format as outlined below. In addition to reviewing this sample exam, it is suggested that you review the past quizzes, the sections in the book, your class notes and the homework.

(4 points each – 5 parts) **Define** the following terms:
a) *relatively prime* polynomials
b) *Galois imaginary*
c) *transposition*
d) *group*
e) *The dihedral group* D_4
f) *A ring*

(2 points each – 5 parts) Answer **True** or **False** - no work required:
a) $x+1$ and $x-1$ are relative prime over all fields.
b) $GF(3, x^5+2x+1)$ contains 242 elements.
c) The permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 6 & 7 & 1 & 8 & 2 & 9 & 5 & 4 \end{pmatrix}$ is even.
d) The negative real numbers under multiplication is a group.
e) The order of the number 2 in the group $(\mathbb{Z}_3, +)$ is 2.
f) The set of 10th roots of unity in $\mathbb{C}$ is a group (under multiplication).
g) Each element $\zeta \neq 0, 1$ in $GF(2, x^5+x^2+1)$ is primitive.

Part III: Computational Problems - there will be a few problems something *like* the following for a total of 40 points.

Completely factor $P(x) = x^3+2x+3$ over $\mathbb{Z}_5$.

Find a greatest common divisor of $P(x) = x^5+4x^3+3x^2+4x+1$ and $Q(x) = x^4+x^3+2x+1$ over $\mathbb{Z}_5$.

Find the remainder when dividing x^{51} by $x+4$ over $\mathbb{Z}_7$.

For which $a \in \mathbb{Z}_5$ is $P(x) = x^3+x+a$ irreducible over $\mathbb{Z}_5$?

Find a greatest common divisor of $P(x) = x^5+4x^3+3x^2+4x+1$ and $Q(x) = x^4+x^3+2x+1$ over $\mathbb{Z}_5$.

Find the inverse of the element $1+2\beta$ in $GF(3, x^2+x+2)$, where β is the associated Galois imaginary.

Find all the primitive elements in $\mathbb{Z}_{19}$

Find all primitive elements in $GF(2, x^4+x+1)$.

Let $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 6 & 7 & 1 & 8 & 2 & 9 & 5 & 4 \end{pmatrix}$ and $\rho = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 3 & 1 & 9 & 8 & 7 & 6 & 5 & 4 \end{pmatrix}$.

a) Express both σ and τ in disjoint cycle notation and as a product of transpositions.
b) Express σ^{-1} and $\sigma\tau$ in disjoint cycle notation.
c) Find the orders and parity of σ, τ, and $\sigma\tau$.
d) Compute σ^{34}.

Consider the following set of matrices:

$$G \equiv GL_2(\mathbb{R}) = \{A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in \mathbb{R} \text{ and } \det(A) \neq 0\}$$

G is a group under matrix multiplication - called the general linear group of 2 by 2 invertible matrices over the real numbers. Why is G closed under multiplication? What is the identity