Peter Cholak and Juan Migliore Math 222 - Sample Final Monday, April 30

The final exam will be Monday, May 7 from 1:45 pm to 3:45 pm in 356 FITZ.

The final exam will cover all the material discussed in class through Chapter 9 (sections 9.1-9.5 – we did not cover isomorphisms of groups in 9.3 but otherwise we covered 9.3) Fair game for the final are all questions (or similar questions) on the previous exams and sample exams, homework, and quizzes. The best way to prepare for the final is to be sure you can do all problems on this sample final, the previous two sample exams, previous two exams, the quizzes and the homework.

The exam will consist of five parts. Part I: Definitions. Part II: True/False. Part III: Computational Problems. Part IV: Known Proofs. Part V: an easy unknown proof. The following are some examples from the latest material.

Part I: Definitions.

Define the following terms:
a) *coset*
b) *cyclic group*
c) *order of a group G*
d) *order of an element g in G*

Part II: True/False.

Answer **True** or **False** - no work required:
a) $\{0, 2\}$ is a coset of $(\mathbf{Z}_6, +)$.
b) D_3 is cyclic.
c) all groups of order 4 are cyclic.
d) The set of 2×2 matrices with entries from the reals numbers is field.
e) No non-zero 2×2 matrix is a zero divisor.
f) The polynomial $x^4 + x + 1$ over $\mathbb{Q}$ is reducible (Hint: Eisenstein's Criterion).

Part III: Computational Problems.

Be sure to know examples of rings, integral domains, fields, groups, etc.

Find all the possible orders of subgroups of D_5. Give examples of subgroups of each of these orders. Compute the index of each of these subgroups in D_5.

Find all the cosets of $\{id, (12)(34)\}$ inside A_4.

Does S_4 have subgroups of the following orders? Why or why not? 2, 3, 4, 5, 6, 7, 8, 9, 10. If so be sure to find a subgroup of that order.

Does S_7 have cyclic subgroups of the following orders? Why or why not? 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12. If so be sure to find a cyclic subgroup of that order.

Consider the subgroup $H = \{Id, (12), (34), (12)(34)\}$ of S_4. Is H cyclic?

Part IV: Proofs. There will be two or three *known* proofs. These will come from the following: