

Peter Cholak and Juan Migliore Math 222 Monday, February 19, 2001

Quiz 3

Be sure to carefully write up your answers. It is suggested that you first write out a draft of your proposed questions and then carefully rewrite that draft to get your final version. You do *not* have to write the answers on this sheet of paper.

Consider the set of integers modulo 10, \mathbb{Z}_{10} .

- (a) Identify the additive inverse of each element in \mathbb{Z}_{10} .
- (b) Identify those elements that have a multiplicative inverse and what their inverses are.

Let n be a positive integer and consider the sum $1 + 2 + 3 + \dots + (n - 1)$ modulo n . Show that if n is odd, this sum is zero in \mathbb{Z}_n and if n is even then the sum is $n/2$ in \mathbb{Z}_n . *Hint:* Use the formula for this sum which we saw when discussing induction.

In \mathbb{Z}_{66} , consider the elements 6, 8, 9, 15, 35 and 55. Identify the one that has a multiplicative inverse in \mathbb{Z}_{66} and find that inverse.

Prove that if p is a prime and $\alpha, \beta \in \sqrt[p]{1}$ and $\alpha \neq 1$ then there exists an integer m such that $\alpha^m = \beta$. (Hints: First, write $\alpha = \zeta^k$ and $\beta = \zeta^r$ where ζ is the first p th root of unity. Second, do k and r have multiplicative inverses in \mathbb{Z}_p ?)