Peter Cholak and Juan Migliore Math 222 Monday, March 19, 2001
Quiz 4

Be sure to carefully write up your answers. It is suggested that you first write out a draft of your proposed questions and then carefully rewrite that draft to get your final version. You do *not* have to write the answers on this sheet of paper.

Find the following examples. You do not have to prove your answers.

1. A ring that is not commutative.
2. A commutative ring that is not unital.
3. A commutative unital ring that is not an integral domain.
4. An integral domain that is not a field.
5. A field that contains the rational numbers $\mathbb{Q}$ and contains $\sqrt{3}$ but is not the whole set of real numbers.

Let E be $\{a + b\sqrt{2}i \mid a, b \in \mathbb{Q}\}$, where i is the complex number with $i^2 = -1$. We know that E is a ring with usual addition and multiplication.
(1) Show that E is a unital ring.
(2) For any nonzero element $x = a + b\sqrt{2}i \in E$, show that $x^{-1} \in E$.
(3) Show that E is a field.

Consider the set $\{0, 2, 4\}$ under the usual addition and multiplication modulo 6. This is a ring (you don't have to check that). Is it a commutative ring? a unital ring? a field? (Hint: write out a times table)

Let $p(x) = 2x^2 + 3x + 4$ and $q(x) = 4x^5 + 4x^4 + 4$. Working in $\mathbb{Z}_5[x]$ find $d(x)$ and $r(x)$ such that $q(x) = d(x)p(x) + r(x)$ and the degree of $r(x)$ is less than the degree of $p(x)$.