Peter Cholak and Juan Migliore Math 222 Friday, March 30, 2001
Quiz 5

Be sure to carefully write up your answers. It is suggested that you first write out a draft of your proposed questions and then carefully rewrite that draft to get your final version. You do *not* have to write the answers on this sheet of paper.

Let F be a field. Show that there exist $a, b \in F$ such that $x^2 + 2$ is a divisor of $x^{43} + ax + b$. (Hint: consider the form of the remainder $r(x)$ when x^{43} is divided by $x^2 + 2$. Do not do the actual division. The degree of $r(x)$ is ??)

Factor $x^3 + 3x + 1$ over $\mathbb{Z}_5$ into irreducible factors.

Consider the Galois Field $F = GF(3, x^2 + x + 2)$. Let α be the associated Galois imaginary.

(a) Show that α is a primitive element in F. Work out the corresponding cyclic table of F.

(b) Find the inverse of each nonzero element in F. *Hint:* Use part (a).

(c) By definition α is one solution to $x^2 + x + 2 = 0$ over $\mathbb{Z}_3$. There should of course be another solution. It is also an element of F. Find this element. Is it a power of α? *Hint:* Use long division or try the other possibilities.

Problem 14 from Section 7.2: Let p be a prime number, and let n be relatively prime to $p^\nu - 1$. Prove that there is exactly one nth root of unity in $GF(p, P(x))$, where $\nu = \deg P(x)$.