Peter Cholak and Juan Migliore Math 222 Wednesday, April 11, 2001
Quiz 6

Be sure to carefully write up your answers. It is suggested that you first write out a draft of your proposed questions and then carefully rewrite that draft to get your final version. You do *not* have to write the answers on this sheet of paper.

Section 7.3, Number 10 (slightly modified) (5 points):

a. For any element $a \in GF(p, P(x))$, let $b \in GF(p, P(x))$ be an element such that $b^{p-1} = a$. Show that then $(kb)^{p-1} = a$ for $1 \leq k \leq p-1$.

b. For any element $a \in GF(p, P(x))$, let $r(a)$ denote the number of distinct elements b in $GF(p, P(x))$ such that $b^{p-1} = a$. Prove that if $a \neq 0$, then $r(a) = 0$ or $p-1$. (You have to show that these are the *only* two possibilities.)

Section 8.2, Problems 35 and 36:

a. (5 points) Prove that if the set S is finite and σ is a function of S into itself, then the following are equivalent (Hint: consider the set $\sigma(S) = \{\sigma(x) \mid x \in S\}$):

   (i) If x_1 and x_2 are distinct elements of S then $\sigma(x_1) \neq \sigma(x_2)$ (i.e. σ is *one-to-one*).

   (ii) If y is any element of S then there is an element x in S such that $y = \sigma(x)$ (i.e. σ is *onto*).

b. (5 points) Show, by means of examples, that when S is an infinite set then neither of the conditions (i) or (ii) necessarily implies the other. Specifically, give an example where

   - (i) is true but (ii) is false;
   - (ii) is true but (i) is false;
   - both (i) and (ii) are true.

(5 points) Let $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 3 & 4 & 1 & 6 & 7 & 5 & 9 & 8 \end{pmatrix}$ and $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 9 & 3 & 4 & 2 & 7 & 5 & 1 & 6 & 8 \end{pmatrix}$.

a. Express σ, τ, σ^{-1} and $\sigma\tau$ in disjoint cycle notation.

b. Find the orders of σ, τ, and $\sigma\tau$.

c. Write σ and τ as a product of transpositions.

d. Find the parity of σ, τ, $\sigma\tau$ and $\sigma\tau^2$.