Math 223: Introduction to Mathematical Reasoning
Spring, 2004

Meeting Time: MWF 1:55 p.m.–2:45 p.m.

Meeting Place: Hayes-Healey 215

Instructor: Matthew Dyer

Office: Hayes-Healey 140

Office hours: Tu 12:00-1:00 p.m, F 12:00–1:00 p.m. or by appointment

Phone: (574) 631-6082

Email: dyer.1@nd.edu

Textbook: Mathematical Thinking (Problem Solving and Proofs) 2nd Edition, by John D'Angelo and Douglas West.

Why this course: Up till now, most of your mathematics courses have likely emphasized examples, computation, and intuitive understanding of mathematics. This course will emphasize careful mathematical arguments. By addressing questions about familiar things like numbers (Are there finite or infinitely many prime numbers? Do all rational numbers have rational square roots?) and sets (What does it mean for a set to have "infinitely many" elements? Do all sets with infinitely many elements have the same size?), we will see how it is that one justifies statements in mathematics. In a nutshell, the subject of this course is numbers, and its goal is to help you understand, invent, and present proofs.

What we'll cover: The material covered falls into roughly three categories (with a fair amount of overlap).

(i) Basic material concerning sets, logic, and methods of proof. Chapters 1–4 of the textbook.

(ii) Algebra and Number Theory (i.e. things relating to integers). Chapters 5–8.

(iii) Analysis (i.e. things relating to real numbers). Chapters 13–15.

As the course unfolds, we might deviate a bit from this plan, but I expect we'll mostly hold to it.

How you will be evaluated:

(i) Homework: Worth 30% of your final grade.

(ii) Two Midterm Exams: Given in class, each worth 20% of your final grade.

(iii) Final Exam: Worth 30% of your final grade.

Grade cutoffs for the major grades (A,B,C,D,F) will be assigned for each exam so you may judge your exam performance.

Exams:

Exam 1: Friday February 20

Exam 2: Friday April 2.

Final Exam: Monday, May 3 at 8 AM. Room TBA.

The in-class exams are intended to examine material covered in class after the preceeding test, but knowledge of earlier material may be required also. The final exam will cover material from the whole course.

Under extreme circumstances, I will consider giving a makeup exam, but you'll need an excuse from whoever it is at ND that officially doles these things out. Also, I reserve the right to shift weights in the final grade so that a makeup exam is less heavily weighted than its regularly scheduled counterpart.

Homework Homework will be assigned and collected (more or less) every Wednesday. I encourage you to collaborate with each other on homework problems. Discussing homework with one another and with me is a good way both to learn the material and gain facility in communicating it to others. In fact for each assignment, you may join forces with *one or two* other students to turn in a single, jointly written set of solutions (with all collaborating students names listed on the top).

Late homework will not be accepted. Regardless of how much you have left to do or how good your reason is, I will only grade the work that you give me at the beginning of class the day it's due. If you think you've got a good reason (e.g. medical emergency or something equally dire) for not finishing some assignment, we can talk about *discounting* it, but there will be no late grading.

Honor Code This course will be conducted under the Honor Code. Copying another student's work in an exam is of course a violation of the code. During exams, where I expect you to work alone, you must direct all questions and comments concerning the exam directly to me. Any reliance on other students or other exams is out of bounds.

Discussion of homework problems with other class members is permitted, as long as your written homework is prepared by those (one to three) students whose name is listed on the assignment and reflects their final understanding of the homework problems. However, direct copying of another student's (or group of student's) written work is a violation of the Honor Code.

General advice for the course

(i) One of the most effective ways of learning mathematics is by solving a range of problems involving the subject matter and skills you are learning. By doing this, you consolidate your understanding of the material and skills, begin to build a useful mathematical intuition for them, and gain experience in problem solving. In this course, with its emphasis on understanding, inventing and presenting proofs, this is especially important. For these reasons (as well as its contribution to your assessment) the homework should be regarded as an integral part of the course.

Additionally, your mathematical understanding is of little use if it cannot be imparted to others, and carefuly writing homework solutions will develop your skills in mathematical communication. You should make every effort to make your written homework solutions as clear as possible. Some points to consider in doing this are that your solutions should be written in complete sentences, with major portions separated into paragraphs, all symbols you use should be defined and you should provide justifications for all major steps you use.

(ii) Please ask me questions, in or out of class. That helps you (you can correct misunderstandings or uncertainties before we move on to new material, which often may build on the foundation you're unsure about), helps the other class members (many of whom may have wanted to ask a similar question) and helps me (by giving me more a more explicit idea about what points may be causing difficulty). The exception to this concerns last minute questions about homework. I give at least a week to complete each homework assignment, and since I expect you to use the full week, I'll be loathe to deal with basic questions about an assignment (like *"how do I start problem 3?"*) the day before it's due.

(iii) I'd be very happy to see you for office hours. If you can't get to the office hours I've set, don't hesitate to contact me to set up an appointment.

(iv) Keep up with the reading in the class. Reading math is a skill that takes time to learn, but the payoff is very high. By learning to read you free yourself to learn on your own terms, independent of professors and courses and schedules (and high tuition!).

(v) (For those who took the last item seriously.) The main thing about reading math books is that you must read slowly and *actively*. In order to make sense of even well-written mathematics, you must often stop to think about what a particular phrase or sentence means, go back to remind yourself of a previous definition or theorem, consider how a statement could and could not be rewritten, or do a pencil and paper computation to convince yourself that some equation is true. You'll find that this approach to reading is infectious. Once you get used to it, you'll tend to approach lectures and newspapers and novels the same way and that all these things become much more fun and edifying once you make yourself an integral part of them.