Math 223 02, Spring 2004 syllabus

Text Mathematical thinking, problem-solving and proofs, John D'Angelo and Douglas West, Prentice-Hall, 2000.

Introduction to Introduction to Mathematical Reasoning absolute value, elementary inequalities such as triangle inequality and arithmetic-geometric mean inequality as introduction to the notion of a proof (preface and pp4-10)

Sets, functions, and numbers (Chapter 1 of text) sets, equality of sets, standard sets (integers, natural numbers, reals, rationals, empty set), equality of sets, subsets, operations on sets (union, intersection, difference, Cartesian product,power set), Venn diagrams, definition of functions, sum and product of real-valued functions, real polynomial functions, identity function, constant functions, bounded real valued functions, monotone real-valued functions, discussion of axioms for real numbers as complete ordered field

Language and Proofs (Chapter 2) mathematical statements, negation and quantification of such, compound statements (conjunction, disjunction, implication), elementary proof techniques (direct proof, contrapositive, proof by contradiction, counterexamples), iff, converse of a statement

Induction (Chapter 3) natural numbers, principle of induction, strong induction, well-ordering of the natural numbers. Many examples of inductive proofs including binomial theorem and existence of at most n zeros of a degree n real polynomial (using factor theorem)

Bijections and cardinality (Chapter 4) Bijections, inverse functions, injections and surjections, composition of functions, associativity of composition, preservation of injections, surjections, bijections under composition, permutations of a set, two-line and cycle notation for permutation of a finite set, cardinality of a finite set and elementary properties thereof, cardinality of sets, countable and uncountable sets, countability of $\mathbf{N} \times \mathbf{N}$, statement (no proof) of Schroder-Bernstein theorem, proof of uncountablity of set of sequences from $\{0, 1\}$

Divisibility (Chapter 6) Divisibility of integers, division algorithm for integers, integer linear combinations, ideals (of integers), theorem that an ideal (in $\mathbf{Z}$) is principal, greatest common divisors and least common multiples, relation between gcd (of two integers) as generator for the ideal of their integer combinations, prime numbers, fundamental theorem of arithmetic, Euclidean algorithm (expression of a gcd as an integer linear combination), infinitude of the set of primes.

Modular Arithmetic (Chapter 7) Relations, reflexive, symmetric, transitive, anti-symmetric properties of a relation, partial and total orders, equivalence relations, fundamental theorem on equivalence relations (equivalence relations and partitions of a set), congruence modulo n, binary operations, addition and multiplication modulo n, the set (commutative ring) of integers modulo n, discussion of existence and computation of a multiplicative inverses modulo n, fact $\mathbf{Z}_p$ is a field for p prime and not an integral domain if p is composite, Chinese remainder theorem, Fermat's little theorem, Wilson's theorem, definition of group, basic examples of groups (integers, additive and multiplicative groups of fields, permutations of a set), very basic properties of group (uniqueness of identity element and inverses, formula for inverse of a product)

The rational numbers (Chapter 8 and Appendix A1-A20) Discussion (without proof) of constructions of $\mathbf{N}$ from set theory and $\mathbf{Z}$ from $\mathbf{N}$, detailed construction of $\mathbf{Q}$ as field of quotients of $\mathbf{Z}$, rational functions (field of quotients of polynomial rings over a field), rational zero's theorem (on zeros of a polynomial with integer coefficients), parameterization of rational points on the unit circle, determination of all Pythagorean triples

The real numbers (Chapter 13) Sups and infs, completeness axiom for $\mathbf{R}$, application to existence of $\sqrt{2}$, Archimeadean property of $\mathbf{R}$, limit of a real sequence, uniquencess of limit, characterization of sups by sequences, monotone convergence theorem, existence of canonical decimal (or k-ary) expansion of a real number, fact every decimal (or k-ary) expansion represents a real number, uncountability of $\mathbf{R}$

Sequences (Chapter 14, 14.1–14.19) Limit of sum, product, quotient sequences, squeeze theorem, Cauchy sequences, subsequence of a sequence, Bolzano-Weierstrass theorem, Cauchy convergence criterion, brief discussion (without proof) of construction of $\mathbf{R}$ from $\mathbf{Q}$ using Cauchy sequences