Name: ______________________________ **Instructor:** Jeffrey Diller

Math 223: Introduction to Mathematical Reasoning
Spring Semester 2004
Exam 1
Friday, February 20

This examination contains 4 problems. Counting the front cover and blank pages, the exam consists of 7 sheets of paper. No calculators allowed.

Scores

Question	Possible	Actual
1	30	
2	30	
3	39	
4	1	
Total	100	

GOOD LUCK

Do each of the following (6 points each).

Define *prime number*.

State the *well-ordering property*.

State the contrapositive of the following assertion: *If* $x_1 \neq x_2$*, then* $f(x_1) \neq f(x_2)$.

Without using words of negation, state the opposite (i.e. negation) of the following assertion: *For all* $x \in A$ *there exists* $b \in B$ *such that* $b > x$.

Define *increasing function*.

Four of the following seven assertions are false. Identify three of them and give counterexamples on this page and (if necessary) the next. If you give counterexamples to more than three statements, I will simply grade the first three. Note that you do not have to justify your counterexamples—only present them. (10 points each)

Let $f : A \to B$ and $g : B \to C$ be functions. If f is injective and g is injective then $g \circ g$ is injective.

Let $f : A \to B$ and $g : B \to C$ be functions. If $g \circ f$ is injective then g is injective.

The function $f : \times \to$ given by $f(a, b) = a + b$ is surjective.

If A, B, C are sets then $(A \cup B) \cap (A \cup C) = A \cup (B \cap C)$

Let $S = \{(a, b) \in \times : 2a + b \leq 5\}$. Then $S \subset [2] \times [2]$.

Let A and B be sets and $f : A \to B$, $g : B \to A$ be injective functions. Then the cardinality of A is the same as that of B.

Let A and B be sets. Suppose that A is a subset of B, but B is not a subset of A. Then the cardinality of A is not the same as the cardinality of B.

On the remaining pages of this exam, do three of the following four problems. If you turn in solutions to all four, I will simply grade the first three. (13 points each)

Let $n \in$ be a natural number whose base 3 decimal expansion is $122_{(3)}$. Give the base 7 decimal expansion of n.

Prove that there are infinitely many prime numbers.

Let A, B, C be sets. Prove that $A - B \subset A - (B \cap C)$

Prove by induction that $\sum_{i=1}^{n} 2n - 1 = n^2$.

How many points should Professor Diller give you for problem 4? (1 point total)