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Math 223: Introduction to Mathematical Reasoning
Spring Semester 2004
Exam 2
Friday, April 3

This examination contains 6 problems. Counting the front cover and blank pages, the exam consists of 7 sheets of paper. No calculators allowed.

Scores

Question	Possible	Actual
1	18	
2	32	
3	10	
4	15	
5	10	
6	15	
Total	100	

GOOD LUCK

Do each of the following (6 points each).

What does the expression '$a \equiv b n$' mean? In particular, what must a, b, and n be for the expression to be meaningful?

Define *relation*.

State Fermat's Little Theorem.

Four of the following seven assertions are false. In the space at the bottom of this page, identify three of them and give counterexamples. Note that you do not have to justify your counterexamples—only present them. (8 points each)

Given $a, b, c \in$ such that $a|bc$ it follows that $a|b$ or $a|c$.

Given $a, b \in$, there exists $m, n \in$ such that $am + bn = \gcd(a, b)$.

Every $\bar{a} \in_5 -\{\bar{0}\}$ has a multiplicative inverse.

If $a, b, k \in$ and k divides an integer combination of a and b, then $k|a$ and $k|b$.

If $2x \equiv 04$, then $x \equiv 04$.

If $a, b, k \in$ are integers such that k divides $a + b$ and a, then k divides b.

Let us say that two integers are related if the difference between the larger and the smaller of the two is prime. Then this defines an equivalence relation on .

Find the closest number $x \in$ to 100 such that

$$x \equiv 12, \quad x \equiv 13, \quad x \equiv 57.$$

Note that x can be either greater or less than 100. (10 points)

Use the Euclidean algorithm to compute $k = \gcd(187, 255)$ and to express k as an integer combination of 187 and 255. (15 points)

Show that if $k|a$ and $k|b$, then k divides any integer combination of a and b.

Given $a, b, n \in$ such that $\gcd(a, b) = 1$, show that $\gcd(na, nb) = n$. (10 points)

Prove that if $a, b \in$ are integers such that $4|(a^2 + b^2)$, then a and b are even. (15 points)