Name: ______________________________ **Instructor:** Jeffrey Diller

Math 223: Introduction to Mathematical Reasoning
Spring Semester 2004
Final Exam
Tuesday, May 4

This examination contains 6 problems. Counting the front cover and blank pages, the exam consists of 9 sheets of paper. No calculators allowed.

Scores

Question	Possible	Actual
1	30	
2	40	
3	10	
4	10	
5	30	
6	30	
Total	150	

GOOD LUCK

State the following (6 points each).

Definition of *convergent sequence*.

Definition of *cauchy sequence*.

Definition of *prime number*.

Bolzano-Weierstrass Theorem.

Definition of *countable set*.

Five of the following eight assertions are false. In the space at the bottom of this page, identify four of them and give counterexamples. Note that you do not have to justify your counterexamples—only present them. (10 points each)

Given $a, b, c \in$ such that $a|bc$ it follows that $a|b$ or $a|c$.

A sequence $\langle x \rangle$ cannot have more than one limit.

If $f : A \to B$, $g : B \to C$ are functions such that $g \circ f$ is injective, then g is injective.

A bounded sequence is convergent.

If S is a set of rational numbers that is bounded above, then the least upper bound of S is a rational number.

If $A \subset B$ and $A \subset C$, then $A \subset B \cap C$.

If $\langle y \rangle$ is a subsequence of $\langle x \rangle$ and $\lim_{n\to\infty} x_n = L$, then $\lim_{n\to\infty} y_n = L$.

If $2x \equiv 24$, then $x \equiv 14$.

Compute the 4–ary (base 4) expansion for 1/5. (10 points)

Prove that is uncountable (10 points).

On the next two pages, do two of the following three problems (15 points each + 5 points extra credit for correct solutions to (b) and (c)).

Prove for every $n \in$ that $\sum_{j=1}^{n} j = \frac{n(n+1)}{2}$.

Let A, B, C be sets. Prove that $A \cap (B \cup C) \subset (A \cap B) \cup C$. Venn diagrams do not constitute a proof!

Let p be a prime number and $n \in$ satisfy $n^2 \equiv 1 p$. Show that $n \equiv \pm 1 p$.

On the next two pages, do two of the following three problems (15 points each + 5 points extra credit if you do (b) and (c) and get them both right).

Show using the definition of limit that $\lim_{n\to\infty} \frac{n-1}{n+1} = 1$.

Suppose that $\langle x \rangle$ is a sequence that converges to a limit L. Show using the definition of limit that $\lim_{n\to\infty} x_n^2 = L^2$.

Let $L \in$ be a given number and $x_1 \in$ be some number larger than L. Then for every $n \in$, let $x_{n+1} = \frac{x_n+L}{2}$. Show that $\lim_{n\to\infty} x_n = L$.

Extra Credit: I won't even look at this unless you've turned in reasonable solutions to the other problems on this exam (10 points). Let $p, q \in$ be natural numbers such that $1 \leq p < q < 10$. What is the maximum number of digits that one must find in the base 10 expansion of p/q before the digits begin to repeat? Justify your answer, *without actually computing any decimal expansions.* Let $k \in$ be larger than 1, and $n \in$ be smaller than $k - 1$. Determine the k-ary expansion of $\frac{n}{k-1}$ and prove that your answer is correct.