Axioms for $\mathbf{R}$

Field Axioms: There are operations $+$ (addition) and $\cdot$ (multiplication) on $\mathbf{R}$ with the following properties.

- $+$ and $\cdot$ are commutative:
$$x + y = y + x, \qquad x \cdot y = y \cdot x.$$
- $+$ and $\cdot$ are associative:
$$x + (y + z) = (x + y) + z, \qquad x \cdot (y \cdot z) = (x \cdot y) \cdot z.$$
- There is an additive identity $0 \in \mathbf{R}$ and a multiplicative identity $1 \in \mathbf{R}$:
$$x + 0 = x, \qquad x \cdot 1 = x.$$
- For each $x \in \mathbf{R}$ there is $y \in \mathbf{R}$ such that $x + y = 0$. If $x \neq 0$, then there is $z \in \mathbf{R}$ such that $xz = 1$. (We write $y = -x$ and $z = 1/x$.)
- Multiplication is distributive over addition:
$$x \cdot (y + z) = x \cdot y + x \cdot z.$$

Order Axioms: There is a relation $<$ on $\mathbf{R}$ with the following properties

- (Trichotomy) For each $x, y \in \mathbf{R}$ exactly one of the following is true: $x < y$, $y < x$, or $x = y$.
- (Transitivity) If $x < y$ and $y < z$, then $x < z$.

Axioms combining arithmetic and order:

- If $x < y$ then $x + z < y + z$.
- If $x, y > 0$ then $x \cdot y > 0$.