Homework 5

From the book: 4.44, 4.47

My problems:

1. Let a, b, c, d be real numbers such that $a < b$ and $c < d$. Show that the closed intervals $[a, b]$ and $[c, d] \subset \mathbf{R}$ have the same cardinality.

2. Let $a, b \in \mathbf{R}$ be positive numbers. Use the Schroeder-Bernstein Theorem to show that the open interval $(0, a)$ has the same cardinality as the closed interval $[0, b] \subset \mathbf{R}$.

3. Prove that if n and m are natural numbers and $f : [n] \to [m]$ is surjective, then $n \geq m$. Suggestion: use induction on n. In the inductive step, there'll be two cases to consider—given $f : [k + 1] \to [m]$, you might have $f(k + 1) = m$ or you might have $f(k + 1) < m$.

Extra credit: 4.48 (6 points for the formula for the bijection, 6 for a correct proof that the function is bijective)

Remark: In problem 4.47 and in my problem 1, you just have to give correct formulas for the bijections and their inverses—you don't have to prove that they're correct. Similarly, in my problem 2, you'll need to define two injections. You just have to give formulas that work—you don't actually have to prove they work.