Math 225: Calculus III Name:______________________________

Exam II *November 1, 1990* Score:______________________________

Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this answer sheet. There are 16 multiple choice questions worth 6 points each. You start with 6 free points.

Find the limit $\lim_{(x,y)\to(1,1)} \frac{x^2-y^2}{x-y}$. 2 0 1 -1 *does not exist*

Let $f(x,y) = y^x$. Compute $f_{xy}(2,e)$. $3e$ 1 e $2e$ 0

A rectangular sheet of metal is expanding. When the width is 2 inches and the length is 3 inches, the width is increasing at the rate of 0.25 in/hr and the length at 0.5 in/hr. At what rate is the area of the rectangle increasing? 1.75 in^2/hr 0.75 in^2/hr 3 in^2/hr 4.5 in^2/hr 6 in^2/hr

Suppose that z is a function of u and v, and that $u = \cos(xy)$ and $v = \sin(x/y)$. If $z_u(-1,0) = 3$ and $z_v(-1,0) = 1$, find z/dy when $x = \pi$ and $y = 1$. π 1 0 $-\pi$ 3π

Which of the following represents the graph of the function $f(x,y) = 12y - y^3 - 3x^2$?

Suppose $w(u,v)$ is a function of u and v and that $w/du = 4u^3v^2 - 2uv^4$, $w/dv = 2u^4v - 4u^2v^3$. The equation $w(u,v) \equiv 0$ defines u as a function of v. Find $\frac{du}{dv}$. $\frac{u(2v^2-u^2)}{v(2u^2-v^2)}$ 0 $\frac{2v(v^2-u^2)}{u(u^4-6v^2)}$ $\frac{v(v^2-6u^2)}{4u(u^2-v^2)}$ $-\frac{2u^3v^2-2uv^3}{2u^4v-4u^2v^3}$

Compute the derivative of $f(x,y) = \ln(x^2y - y^2 - 2)$ in the direction $3 \subset +4 \supset$ at the point $(2,1)$. 4 $12 \subset +8 \supset$ $4 \subset +2 \supset$ 20 *does not exist*

Find the direction in which the function $f(x,y) = x^3 - y^2x - 2xy^2$ increases most rapidly at the point $(1,-1)$. $\supset$ $\subset 3x^2 \subset -2xy \supset$ $(x^2-y) \subset -x \supset$ $2 \subset - \supset$

Determine the equation of the tangent plane to the paraboloid $z = 9 - 4x^2 - y^2$ at the point $(1,1,4)$. $8x + 2y + z = 14$ $x + y + 4z = 18$ $4x + 2y + z = 10$ $x + 8y + 4z = 25$ $x + 4y + z = 9$

Compute the differential of the function $f(x,y,z) = x\cos(\frac{y}{z})$. $\cos(\frac{y}{z})\,dx - \frac{x}{z}\sin(\frac{y}{z})\,dy + \frac{xy}{z^2}\sin(\frac{y}{z})\,dz$ $\cos(\frac{y}{z}) + \frac{x}{z}\sin(\frac{y}{z}) + \frac{xy}{z^2}\sin(\frac{y}{z})$ $\cos(\frac{y}{z})\,dx - \sin(\frac{y}{z})\,dy - \sin(\frac{y}{z})\,dz$ $\sin(\frac{y}{z}) - \frac{x}{z}\cos(\frac{y}{z}) + \frac{xy}{z^2}\sin(\frac{y}{z})$ $\sin(\frac{y}{z})\,dx - \frac{y}{z}\cos(\frac{x}{z})\,dy$

Determine the critical points of the function $f(x,y) = 3x^2 - 3xy^2 + 2y^3$. $(0,0)$, $(2,2)$ $(0,0)$, $(2,0)$, $(2,2)$ $(0,0)$, $(2,0)$ $(0,0)$, $(2,2)$, $(-2,2)$ $(0,0)$, $(2,0)$, $(2,2)$, $(-2,2)$

The function $f(x,y) = 2x^3 - 6x + 2y^3 - 3xy^2$ has a critical point at $(\sqrt{2}, \sqrt{2})$. Use the Second Partials Test to determine which of the following is true at this point: *f has a relative minimum* *f has a relative maximum* *f has a saddle point* *test is inconclusive* *none of the above*

Find the maximum of the function $f(x, y) = y^3 + 3x^2y$ in the region $x^2 + y^2 \leq 4$. $8\sqrt{2}$ 8 0 16 $16\sqrt{2}$

Determine which of the following sets of equations must be solved to find the extreme values of the function $f(x, y) = x^3y + y^2x + xy^4$ subject to the constraint $x^4 + 4xy + y^4 = 4$. $3x^2y + y^2 + y^4 = \lambda(4x^3 + 4y)$
$x^3 + 2xy + 4xy^3 = \lambda(4x + 4y^3)$
$x^4 + 4xy + y^4 = 4$

$3x^2y + y^2 + y^4 = \lambda(4x^3 + 4y^3)$
$x^3 + 2xy + 4xy^3 = \lambda(4x^3 + 4y^3)$
$4x^3 + 4y + 4y^3 = 0$

$3x^2y + y^2 + y^4 = \lambda(4x^3 + 4y + 4y^3)$
$3x^2 + 2y + 4y^3 = \lambda(4x + 4y^3)$
$x^4 + 4xy + y^4 = 4$

$3x^2y + y^2 + y^4 = \lambda(4x^3 + 4y^3)$
$x^3 + 2xy + 4xy^3 = \lambda(4x^3 + 4y^3)$

$3x^2y + y^2 + y^4 = \lambda(4x$
$x^3 + 2xy + 4xy^3 = \lambda(x$

Let R be the triangle in the plane with vertices at the points $(0, 0)$, $(0, 1)$, and $(1, 1)$. Compute the double integral ${}_R x^2y\, dA$. $\frac{1}{10}$ $\frac{1}{15}$ $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{5}$

Reverse the order of integration of the iterated integral $\int_0^1 \int_{\sqrt{y}}^1 f(x, y)$. $\int_0^1 \int_0^{x^2} f(x, y)$ $\int_1^{\sqrt{x}} \int_0^1 f(x, y)$ $\int_0^1 \int_{\sqrt{x}}^1 f(x, y)$ $\int_{\sqrt{y}}^1 \int_0^1 f(x, y)$ $\int_0^1 \int_{x^2}^1 f(x, y)$