Math 225: Calculus III Name:______________________________

Exam III *December 6, 1990* Score:______________________________

Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this answer sheet. There are 16 multiple choice questions worth 6 points each. You start with 6 free points.

Evaluate ${}_R x\,dA$ where R is the semi-annular region in the first quadrant between the circles of radius 1 and radius 3. $\frac{26}{3}$ $\frac{31}{3}$ 9 $\frac{25}{3}$ 10

Determine which of the following integrals gives the surface area of the portion of the graph of $f(x,y) = e^x\cos(y)$ lying over a region R. ${}_R\sqrt{1+e^{2x}}\,dA$ ${}_R\sqrt{1+e^x\cos(y)-e^x\sin(y)}\,dA$ ${}_R e^x\cos(y)\,dA$ ${}_R e^x\,dA$ ${}_R 1+e^x\,dA$

Let D be the solid bounded by the planes $x+y+z=2$, $y=0$, $y=x$, and $z=0$. Find the integral that gives the volume of D. $\int_0^1\int_y^{2-y}\int_0^{2-x-y} 1\,dz\,dx\,dy$ $\int_0^2\int_0^x\int_0^{2-x-y} 1\,dz\,dy\,dx$ $\int_0^2\int_0^x\int_0^{x+y+z} 1\,dz\,dy\,dx$ $\int_0^1\int_y^2\int_0^{2-y} 1\,dz\,dx\,dy$ $\int_0^1\int_0^{x-y}\int_0^{2-x-y} 1\,dz\,dy\,dx$

Let D be the portion of the ball $x^2+y^2+z^2\le 1$ cut out by the cylinder $x^2+y^2=x$. Write ${}_D xyz\,dV$ as an iterated integral in cylindrical coordinates. $\int_0^\pi\int_0^{\cos(\theta)}\int_{-\sqrt{1-r^2}}^{\sqrt{1-r^2}} zr^3\cos(\theta)\sin(\theta)\,dz\,dr\,d\theta$ $\int_0^{2\pi}\int_0^1\int_{-\sqrt{1-r^2}}^{\sqrt{1-r^2}} zr^2\cos(\theta)\sin(\theta)\,dz\,dr\,d$ $\int_0^{2\pi}\int_0^{\cos(\theta)}\int_0^{\sqrt{1-r^2}} zr^2\cos(\theta)\sin(\theta)\,dz\,dr\,d\theta$ $\int_0^\pi\int_0^{r\cos(\theta)}\int_0^{\sqrt{1-r^2}} zr^3\cos(\theta)\sin(\theta)\,dz\,dr\,d\theta$ $\int_0^{2\pi}\int_0^1\int_0^{\sqrt{1-r^2}} r\cos(\theta)\sin(\theta)\,dz\,dr\,d\theta$

Let D be the solid region bounded above by the sphere $x^2+y^2+z^2=36$ and below by the upper nappe of the cone $3(x^2+y^2)=z^2$. Write ${}_D(x^2+y^2+z^2)^{3/2}\,dV$ as an iterated integral in spherical coordinates. $\int_0^{2\pi}\int_0^{\pi/6}\int_0^6 \rho^5\sin(\phi)\,d\rho\,d\phi\,d\theta$ $\int_0^{2\pi}\int_0^{\pi/3}\int_0^6 \rho^2\sin(\phi)\,d\rho\,d\phi\,d\theta$ $\int_0^{\pi}\int_0^{\pi}\int_0^6 \rho^{3/2}\sin(\phi)\,d\rho\,d\phi\,d\theta$ $\int_0^{2\pi}\int_0^{\pi/2}\int_0^{36} \rho^3\sin(\phi)\,d\rho\,d\phi\,d\theta$ $\int_0^{2\pi}\int_0^{\pi/4}\int_0^{36} \rho^4\sin(\phi)\,d\rho\,d\phi\,d\theta$

Let D be the portion of the solid cylinder $x^2+y^2\le 1$ below $z=2$ in the first octant. Suppose the density at a point in D is twice the distance of the point to the z-axis. Compute the center of gravity of D, assuming its total mass is $\frac{2\pi}{3}$. (Use symmetry.) $(\frac{3}{2\pi},\frac{3}{2\pi},1)$ $(\frac{3}{2},\frac{3}{2},1)$ $(1,1,\frac{1}{2})$ $(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},1)$ $(0,0,1)$

Let R be the region of points (x,y) which satisfy $\frac{1}{2}x\le y\le 2x$, $1\le xy\le 2$. Using the change of variables $u=xy$, $v=\frac{y}{x}$ transform ${}_R xy\,dA$ into an iterated integral in the uv-plane. (Hint: You do not have to solve for x and y.) $\int_{1/2}^2\int_1^2 \frac{u}{2v}\,du\,dv$ $\int_{1/2}^2\int_1^2 uv\,du\,dv$ $\int_{1/2}^2\int_1^2 2uv^2\,du\,dv$ $\int_{x/2}^{2x}\int_1^2 u\,du\,dv$ $\int_1^2\int_{x/2}^{2x}\left(uv+\frac{u}{v}\right)\,du\,dv$

Find a function $f(x,y,z)$ such that $f = (y-z\sin(x))\subset+(x-z)\supset+(\cos(x)-y+2z)$ and then evaluate $f(\pi,2,-1)-f(0,0,0)$. $2\pi+4$ $\pi+2$ 0 $2\pi+3$ 1

Let C be the curve parameterized by $(t)=e^t\subset+e^{-t}\supset+$, $-1\le t\le 1$. Find the integral that gives the value of the line integral $\int_C \frac{x^2+y^2}{z}\,ds$. $\int_{-1}^1\left(e^{2t}+e^{-2t}\right)^{3/2}\,dt$ $\int_{-1}^1\left(e^{2t}+e^{-2t}\right)\,dt$ $\int_{-1}^1\sqrt{1+e^{2t}+e^{-2t}}\,dt$ $\int_{-1}^1\sqrt{e^{2t}+e^{-2t}}\,dt$ $\int_{-1}^1 2e^t\left(1+e^{2t}+e^{-2t}\right)^{3/2}\,dt$

Let $=xy\subset+yz\supset$ and let C be the curve parameterized by $(t)=t\subset+t^2\supset+t^3$, $0\le t\le 1$. Find the value of $\int_C d$. $\frac{15}{28}$ $\frac{3}{4}$ $\frac{7}{24}$ $\frac{5}{6}$ $\frac{5}{12}$

Let C be a smooth curve from $(1,1,0)$ to $(0,0,1)$. Compute $\int_C \cos(yz)\,dx - xz\sin(yz)\,dy - xy\sin(yz)\,dz$. -1 0 $\cos(1)$ 2 $-3\cos(1)$

Let C be the counterclockwise path around the perimeter of the triangle with vertices $(0,0)$, $(2,0)$, and $(2,1)$. By Green's Theorem, the line integral $\int_C e^{x^2}\,dx+e^{y^2}\,dy$. equals: 0 $\frac{e}{2}$ $\int_0^2\int_0^{2x} e^{x^2}+e^{y^2}$ $\int_0^2\int_0^{2x} 2xe^{x^2}+2ye^{y^2}$ $\int_0^2\int_0^{2x} 2xe^{x^2}\subset+2ye^{y^2}\supset$

Let Σ be the upper hemisphere $z=\sqrt{1-x^2-y^2}$. The surface integral ${}_\Sigma z^2\,dS$ equals: $\frac{2\pi}{3}$ 2π $\frac{2}{3}$ $\frac{1}{3}$ $\frac{\pi}{3}$

Let $=x^2y\subset-z^2\supset+y$. Compute $\div$. $2xy$ $2xy-2z+1$ $2xy\subset-2z\supset+$ $2xy+x^2-2z+1$ $(2xy+x^2)\subset-2z\supset$

Let $=x^3\subset+y^3\supset+z^3$. Compute . $3x^2+3y^2+3z^2$ $3x^2\subset+3y^2\supset+3z^2$ $(3y^2-3x^2)$ $6x+6y+6z$

Let $f(x,y,z)$ be a function and let (x,y,z) be a vector field. Determine which of the following expressions is **not** defined. $\div$ $\div$ $\div f$ f $\div$