Math 225: Calculus III Name:______________________________

Final Exam *December 19, 1990* Score:______________________________

Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this answer sheet. There are 25 multiple choice questions worth 6 points each.

A vector is perpendicular to a vector $\underline{if} = 0 \times = 0 + = 0 = \underline{c}, somescalar c(\underline{\times}) = 0$

Compute the projection of $= 2 \subset +3 \supset +5$ onto $= \subset +2 \supset +2$. $2 \subset +4 \supset +4$ $2 \subset +3 \supset +5$ $2 \subset +6 \supset +10$ $\subset +\frac{3}{2} \supset +\frac{5}{2}$ $\frac{1}{2} \subset + \supset +$

Let $= \subset + \supset +$ and $= 2 \subset +3 \supset +4$. Compute $\times$ ı- 2ȷ+ 2ı+ 3ȷ+ 4ı+ 2ȷ+ 3-ı+ 2ȷ- 3

Determine the symmetric equations of the line through the points $(-1,0,3)$ and $(2,1,-1)$. $\frac{x+1}{3} = y = \frac{z-3}{-4}$ $\frac{x+1}{3} = y, \quad z = 3$ $x = -1, \quad y = \frac{z+1}{-4}$ $\frac{x-2}{-1} = \frac{z+1}{3}, \quad y = 1$ $\frac{x-2}{-1} = y - 1 = \frac{z+1}{3}$

Determine the equation of the plane containing the points $(0,0,0)$, $(1,2,3)$ and $(4,5,6)$. $x - 2y + z = 0$ $2x - y = 0$ $5x - 4y = 0$ $4x + y - 2z = 0$ $4x - 2y - z = 0$

Find $\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$. *Does not exist* 1 $\frac{1}{2}$ 0 -1

Suppose the motion of a particle is described by $(t) = e^t \subset +e^{-t} \supset +\frac{1}{\sqrt{2}}t^2$, $t \geq 0$. Find the particle's speed at time $t = 1$. 3.086 4.500 3.765 2.021 2.920

Suppose a particle moves along the path $(t) = \cos(t^2) \subset +\sin(t^2) \supset +t^2$, $0 \leq t \leq \sqrt{2\pi}$. Find the total distance travelled by the particle. 8.886 6.283 4.443 2.507 12.566

A 1 meter cube is being compressed. After 2 seconds, its height is 99 cm and is decreasing at a rate of $1\frac{\text{cm}}{\text{sec}}$ while its depth and width are 101 cm and are increasing at a rate of $0.5\frac{\text{cm}}{\text{sec}}$. Find the rate of change in the volume of the cube after 2 seconds of being compressed. $-202\frac{\text{cm}^3}{\text{sec}}$ $0\frac{\text{cm}^3}{\text{sec}}$ $-99\frac{\text{cm}^3}{\text{sec}}$ $-9801\frac{\text{cm}^3}{\text{sec}}$ $-5\frac{\text{cm}^3}{\text{sec}}$

Let $f(x,y,z) = x^2y + y^2z + z^2x$. Find the derivative of f in the direction of the vector $\sqrt{2} \subset + \supset +$ at the point $(0,1,2)$. $\frac{5}{2} + 2\sqrt{2}$ $4 \subset +4 \supset +$ $\frac{3}{2} + \sqrt{2}$ $6 + \frac{\sqrt{2}}{2}$ $\frac{\sqrt{2}}{2} \subset +\frac{1}{2} \supset +\frac{1}{2}$

Let $f(x,y,z) = xyz^2 + 2x^2y$. Find the direction of maximum increase in f at the point $(-1,-2,1)$. $6 \subset + \supset +4$ $- \subset -2 \supset - 2$ $\subset - \supset - 4$ $\subset -2 \supset -$ $-4 \subset + \supset +2$

Let $f(x,y) = x^3 + xy + y^2$. Determine which of the following statements is true. f has a relative minimum at $(\frac{1}{6}, -\frac{1}{12})$. f has a relative maximum at $(\frac{1}{6}, -\frac{1}{12})$. f has a saddle point at $(\frac{1}{6}, -\frac{1}{12})$. $(\frac{1}{6}, -\frac{1}{12})$ is **not** a critical point of f *None of the above*

Find the minimum of $f(x,y) = x(1 - e^y)$ on the unit square $0 \leq x \leq 1$, $0 \leq y \leq 1$. -1.718 0 -1 -0.718 -2.718

Find the maximum of $f(x,y) = 2y^3 + 6x^2y$ subject to the constraint $x^2 + y^2 = 4$. $16\sqrt{2}$ 18 $18\sqrt{3}$ 16 24

Evaluate $\int_0^2 \int_1^{1-x^2} xy$. $\frac{4}{3}$ $\frac{1}{3}$ 0 1 $\frac{2}{3}$

Reverse the order of integration in the iterated integral $\int_{-1}^1 \int_{|x|}^1 \cos(y^{3/2})$. $\int_0^1 \int_{-y}^y \cos(y^{3/2})$ $\int_{|x|}^1 \int_{-1}^1 \cos(y^{3/2})$ $\int_{-1}^1 \int_0^{|y|} \cos(y^{3/2})$ $\int_0^1 \int_0^y \cos(y^{3/2})$ $\int_{-1}^1 \int_{-y}^0 \cos(y^{3/2})$

Let $= xy^3 \subset +yz^3 \supset +zx^3$. Compute . $-3yz^2 \subset -3x^2z \supset -3xy^2$ $x^3 + y^3 + z^3$ $x^3 \subset -y^3 \supset +z^3$ $-3xy^2 \subset +3yz^2 \supset -3zx^2$ 0

Which of the following integrals gives the volume of the solid bounded by the planes $x + y - z = 0$, $x - y + z = 0$, $x = 0$, $y = 0$, and $x + y = 2$. $\int_0^2 \int_0^{2-x} \int_{y-x}^{y+x} 1\,dz\,dy\,dx$ $\int_0^2 \int_0^{2-x} \int_0^{2-y-x} 1\,dz\,dy\,dx$ $\int_0^2 \int_{y-2}^{2-y} \int_{y-x}^{y+x} 1\,dz\,dx\,dy$ $\int_0^2 \int_0^{2-y} \int_0^{y+x} 1\,dz\,dy\,dx$ $\int_0^2 \int_{2-x}^2 \int_{y-x}^2 1\,dz\,dy\,dx$

Let D be the solid in the first octant that lies below the plane $z = 1$ and is bounded by $x^2 + y^2 = 9$. Write ${}_D xyz\,dV$ in cylindrical coordinates. $\int_0^{\pi/2} \int_0^3 \int_0^1 zr^3 \cos(\theta)\sin(\theta)\,dz$ $\int_0^3 \int_0^\pi \int_0^1 zr^2 \cos(\theta)\sin(\theta)\,dz$ $\int_0^{2\pi} \int_0^1 \int_0^3 zr^2 \cos(\theta)\sin(\theta)\,dz$ $\int_0^{\pi/2} \int_0^3 \int_0^1 zr^3 \cos(\theta)\sin^2(\theta)\,dz$ $\int_0^{\pi/2} \int_0^1 \int_0^3 zr^2 \cos(\theta)\sin(\theta)\,dz$

Compute ${}_D(x^2 + y^2)\,dV$ where D is the unit sphere $x^2 + y^2 + z^2 = 1$. $\frac{8\pi}{15}$ $\frac{\pi^2}{4}$ $\frac{3\pi}{5}$ $\frac{4\pi^2}{3}$ $\frac{4\pi}{5}$

Let C be the curve $(t) = t \subset +t^2 \supset$, $0 \leq t \leq 1$. Compute $\int_C x^3\,dx + xy\,dy$. $\frac{13}{20}$ $\frac{1}{2}$ $\frac{7}{12}$ $\frac{3}{2}$ $\frac{14}{15}$

Let Σ be the portion of the plane $3x + 2y + z = 0$ under the rectangle $0 \leq x \leq 2$, $0 \leq y \leq 3$ in the xy-plane. Compute ${}_\Sigma z^2 - 12xy\,dS$. $144\sqrt{14}$ 144 $288\sqrt{2}$ $288\sqrt{7}$ 288

Let $= xe^{xy} \subset -ye^{xy} \supset +z$, let Σ be the sphere $x^2 + y^2 + z^2 = 4$, and let be the outward normal vector to Σ. Evaluate the flux integral ${}_\Sigma\,dS$. $\frac{32\pi}{3}$ $\frac{4\pi}{3}$ $\frac{8\pi}{3}$ $\frac{16\pi}{3}$ 0

Let C be the intersection of the paraboloid $x^2 + y^2 + z = 2$ and the plane $x + y + z = 1$. Compute $\int_C x^2\,dx + y^2\,dy + z^2\,dz$. 0 $\frac{\pi}{3}$ $\frac{2\pi}{3}$ $\frac{4\pi}{3}$ $\frac{8\pi}{3}$

Let $= -z^2 \subset + e^z \supset + (e^z y - 2xz)$ and let $(t) = t \subset + t^2 \supset + t^3$, $0 \leq t \leq 1$. Evaluate $\int_C d$. $e - 1$ 1 0 $e - e^{-1}$ $2e - 1$