Math 225: Calculus III Name:________________________

Exam II *October 31, 1991* Score:________________________

Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this answer sheet. There are 15 multiple choice questions worth 6 points each. You start with 10 points.

Determine which of the following shaded regions corresponds to the domain of the function

$f(x,y) = \frac{\sqrt{1-(x^2+y^2)}}{xy}$.

Determine which function below has level curves of the form:

$f(x,y) = \frac{y}{3x^2}$ $f(x,y) = x^2 + 3y^2$ $f(x,y) = 3x^2y$ $f(x,y) = x^2 - 3y^2$ $f(x,y) = y - 3x^2$

Determine which function below has a graph like the following:

$f(x,y) = x^3 - x^2 - y^2$ $f(x,y) = x - 1 - y$ $f(x,y) = x^2 - x - y$ $f(x,y) = y - x^2 + x$ $f(x,y) = y^2 - x^3 + x$

Compute the limit $\lim_{(x,y)\to(1,1)}[\ln(x^2-y^2) - \ln(x-y)]$.

$\ln(2)$ 0 1 ∞ *does not exist*

If $f(x,y,z) = \sin(xy)e^{-z} + e^y + x^2$, compute $f_{xz}(1,\pi,0)$.

π 0 1 2 $-\pi/e$

Let $z = e^{-x^2-y^2}$, $x = uv$, and $y = u^2 - v^2$. Compute $\frac{\partial z}{\partial v}$.

$e^{-x^2-y^2}(-2ux+4yv)$ $e^{-u^2+v^2}(-2u^2v+4v)$ $e^{-x^2-y^2}(-2x-2y)$ $e^{-u^2v^2}(-2u^2v+4v(u^2-v^2))$ $e^{-u^4-v^4}(-2x-2y)$

A particle moves along a smooth path in the plane. At time $t = 1$ the particle is at the point $(3,-4)$ and its velocity is $\doteq 2 \subset +3 \supset$. At what rate is the distance of the particle to the origin changing at that moment?

decreasing 1.20 *increasing* 1.20 *decreasing* 3.74 *increasing* 3.74 *neither increasing nor decreasing at* $t = 1$

Find the derivative of $f(x,y,z) = x^3y - 2y^2z$ in the direction of the vector $= - \subset + \supset +2$ at the point $(1,1,1)$.

-4.08 0.00 -10.0 -0.82 2.96

Compute the gradient of the function $f(x,y,z) = \cos(xy) - z^2$.

$-y\sin(xy) \subset -x\sin(xy) \supset -2z$ $-y\sin(xy) \subset +x\cos(xy) \supset +2z$ $-y\sin(xy) - x\sin(xy) - 2z$ $-y\sin(xy) + x\cos(xy) + 2z$ $-\sin(xy) - 2z$

Find the direction in which the function $f(x,y) = y^3x^2 + x$ increases most rapidly at the point $(2,1)$.

$5 \subset +12 \supset$ $3 \subset +3 \supset$ $4 \subset +2 \supset$ 17 13

Determine the equation of the plane tangent to the graph of $f(x,y) = 2x^4 - 3xy^2 + y^3$ at the point $(1,1,0)$.

$5x - 3y - z = 2$ $x - 3y - z = -2$ $8x - 6y - z = 2$ $4x - 6y + z = -2$ $5x - 3y = 2$

Find all the critical points of the function $f(x,y) = 14x^3 - 21x^2y + 5y^3 + 24y + 1$.

$(-2,-2), (2,2)$ $(0,1.26), (0,-1.26), (-2,-2), (2,2)$ $(-2,-2), (-2,2), (2,-2), (2,2), (0,1.26), (0,-1.26),$ $(-2,2), (2,-2)$ $(0,1.26), (0,-1.26)$

Choose the statement below that is **true** about the function $f(x,y) = e^{-x}(x^2+y^2)$.

f has a saddle point at $(2,0)$ *f has a local maximum at* $(2,0)$ *f has a local minimum at* $(2,0)$ $(2,0)$ *is* **not** *a critical point of f* *none of the above*

Find the minimum of $f(x,y) = x^2 - 6xy + y^3$ on the rectangle $0 \le x \le 20$, $0 \le y \le 10$.

-108 -106 -120 -200 0

Find a point on the curve $13x^2 + 10xy + 13y^2 = 4$ where the value of the function $f(x,y) = x^2y^2$ is greater than or equal to its value at any other point on the curve.

$(\frac{1}{2}, -\frac{1}{2})$ $(\frac{1}{3}, \frac{1}{3})$ $(-1,1)$ $(1, -\frac{1}{\sqrt{5}})$ $(\frac{2}{\sqrt{13}}, -\frac{2}{\sqrt{13}})$