Math 225: Calculus III Name:____________________________

Exam III *December 5, 1991* Score:____________________________

Record your answers to the multiple choice problems by placing an × through one letter for each problem on this answer sheet. There are 15 multiple choice questions worth 6 points each. You start with 10 points.

Compute $\int_0^1 \int_0^x e^{x^2}\, dy\, dx$. $(e-1)/2$ e 0 $\frac{1}{2} - e$ 1

Reverse the order of integration in the integral $\int_1^2 \int_1^{x^3} f(x,y)\, dy\, dx$ $\int_1^8 \int_{y^{1/3}}^2 f(x,y)\, dx\, dy$ $\int_1^{x^3} \int_1^2 f(x,y)\, dx\, dy$ $\int_1^2 \int_{x^3}^1 f(x,y)\, dx\, dy$ $\int_0^8 \int_{y^3}^2 f(x,y)\, dx\, dy$ $\int_0^8 \int_0^{y^{1/3}} f(x,y)\, dx\, dy$

The area of the region bounded by $y = 9 - x^2$ and the x-axis is 36. Find the centroid of this region.

$(0, 3.6)$ $(0, 4.0)$ $(0, 2.4)$ $(0, 3.2)$ $(0, 2.8)$

Compute the area of the region that lies inside the cardioid $r = 1 - \cos(\theta)$ in the first quadrant.

$\frac{3\pi}{8} - 1$ $\frac{\pi}{2}$ $\frac{5\pi}{8} - \frac{1}{2}$ $\frac{7\pi}{16} - \frac{1}{4}$ $\frac{9\pi}{16}$

Let D be the solid in the first octant below the plane $4x + 4y + z = 4$. Suppose the density of D is given by $\delta(x,y,z) = 4 - z$. Which of the following integrals gives the total mass of D.

$\int_0^1 \int_0^{1-x} \int_0^{4-4x-4y} 4-z\, dz\, dy\, dx$ $\int_0^4 \int_0^{4-z} \int_0^{4-4x-4y} 1\, dz\, dy\, dx$ $\int_0^1 \int_0^{1-x} \int_0^{4-z} 4-4x-4y\, dz\, dy\, dx$ $\int_0^1 \int_0^{4-x} \int_0^{4-4y} 4-z\, dz\, dy\, dx$ $\int_0^4 \int_0^{4-x} \int_0^{4-4x-4y} 4-z\, dz\, dy\, dx$

Find the volume of the solid bounded above by $z = 4 - x^2 - y^2$, below by the xy-plane, and on the sides by $x^2 + y^2 = 1$. $7\pi/2$ $22\pi/3$ 4π $9\pi/2$ $25\pi/3$

Let D be the solid bounded above by $\rho = 2\sin(\phi)$ and below by $\phi = \pi/2$ in spherical coordinates. Compute ${}_D z\, dV$.

$4\pi/3$ $2\pi/3$ π 2π $\pi^2/2$

Let R be the region bounded by the lines

$$\begin{aligned} x+y=1, &\quad x+y=2 \\ x-2y=-2, &\quad x-2y=1 \end{aligned}$$

Determine which of the following integrals gives the value of ${}_R xy\,dA$ after the substitution $u = x+y$, $v = x-2y$. $\int_{-2}^{1}\int_{1}^{2}\frac{1}{27}(u-v)(2u+v)\,du\,dv$ $\int_{-2}^{1}\int_{1}^{2}\frac{1}{9}(v-u)(2u+v)\,du\,dv$ $\int_{-2}^{1}\int_{1}^{2}(x+y)(x-2y)\,dx\,dy$ $\int_{1}^{2}\int_{-2}^{1}\frac{1}{3}(x+y)(2y-x)\,dy\,dx$ $\int_{1}^{2}\int_{-2}^{1}\frac{1}{3}uv\,du\,dv$

Determine which of the following represents the vector field $(x,y) = -\frac{y}{5} \subset +\frac{x}{5} \supset$.

Let $(x,y) = x^2(x+y) \subset +y^2(y+z) \supset +z^2(x+z)$. Compute $\div$. $3(x^2+y^2+z^2)+2(xy+yz+xz)$ $(3x^2+2xy) \subset +(3y^2+2yz) \supset +(3z^2+2xz)$ $xyz(3x+2y)(3y+2z)(3z+2x)$ $-(y^2 \subset +z^2 \supset +x^2)$ 0

Let $(x,y) = e^x\cos(y) \subset +e^{-x}\sin(y) \supset +$. Compute $(e^x-e^{-x})\sin(y)$ $(e^x+e^{-x})\cos(y)$ $(e^x+e^{-x})\sin(y)$ $e^x\cos(y) \subset +e^{-x}\cos(y) \supset$

Calculate the line integral $\int_C y\,ds$ where C is the curve parameterized by $(t) = t \subset +2t \supset +t^2$, $0 \le t \le 1$. 2.637 1.905 3.872 2.733 3.223

Let C be parameterized by $(t) = (t^3-1) \subset +t^2 \supset +t$, $0 \le t \le 1$.

Evaluate $\int_C 1\,dx + x\,dy + y\,dz$.

0.733 0.5 0.333 1.2 0.822

Let C be parameterized by $(t) = e^{\cos(t)} \subset +e^{\sin(t)} \supset$, $0 \le t \le \pi$, and let $f(x,y) = \sqrt{x^2+y^2}$. Calculate $\int_C (f)\cdot d$. $\sqrt{e^{-2}+1}-\sqrt{e^2+1}$ $\sqrt{e^{-2}+e^2}-\sqrt{2}$ 0 $e^{-1}-e$ $2\sqrt{e^{-2}+1}$

Let C be the boundary of the upper half of a circle of radius 1. Use Green's Theorem to evaluate $\int_C (x^2+y^2)\,dx + xy\,dy$.

$-2/3$ $-1/3$ $-1/2$ $-3/4$ 0