Math 225: Calculus III Name:____________________________

Exam III *Apr. 14, 1994* Section:____________________________

Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this answer sheet. There are 15 multiple choice questions worth 6 points each. You start with 10 points.

Find the area of the region enclosed by the curve $r = \sqrt{3 - 2\sin(2\theta)}$.

3π 2π $5\pi/3$ $7\pi/3\pi$ $8\pi/3$

Evaluate $\int_0^1 \int_0^{\sqrt{x}} \int_0^y e^{-x^2}\, dz\, dy\, dx$ $(1 - e^{-1})/4$ $1/4$ $e^{-1}/2$ $(e-1)/2$ $(e^{-1} - 1)/2$

Find the volume of the solid region in the first octant bounded by the plane $x + y = 1$ and the surface $z = \sin(\pi x)$. $1/\pi$ π $1/2$ $\pi/2$ 1

Let R be the region defined by $1 \le x \le 2$, $1 \le xy \le 2$. Using the substitution $u = x$, $v = xy$, transform ${}_R y\, dA$ into an iterated integral in the uv-plane. $\int_1^2 \int_1^2 \frac{v}{u^2}\, du\, dv$ $\int_1^2 \int_1^{2/u} \frac{v}{u}\, du\, dv$ $\int_1^2 \int_1^{2v/u} \frac{1}{u^2}\, du\, dv$ $\int_1^2 \int_1^2 v\, du\, dv$ $\int_1^2 \int_1^2 \frac{v}{u}\, du\, dv$

Let D be the solid region below the paraboloid $z = 4 - x^2 - y^2$ and above the xy-plane. The density of D is given by $\delta(x, y, z) = 4 - z$. Given that the total mass of D is $64\pi/3$, compute the center of mass of D. $(0, 0, 1)$ $(0, 0, 7/8)$ $(0, 0, 3/4)$ $(0, 0, 5/4)$ $(0, 0, 3/2)$

Find the volume of the solid region between the cone $z^2 = 4(x^2 + y^2)$ and the paraboloid $z = 1 + x^2 + y^2$.

$\pi/6$ π $3\pi/4$ $\pi/2$ $\pi/3$

Find the average value of $f(x, y, z) = (x^2 + y^2 + z^2)^{1/2}$ over the solid unit ball, $x^2 + y^2 + z^2 \le 1$. $3/4$ $1/2$ $5/8$ $7/8$ $2/3$

Find the vector field represented in the following plot:

$\frac{1}{2\sqrt{x^2+1}} \subset +\frac{x}{2\sqrt{x^2+1}} \supset \frac{x}{2\sqrt{x^2+y^4}} \subset +\frac{y^2}{2\sqrt{x^2+y^4}} \supset \frac{x}{2\sqrt{x^2+1}} \subset -\frac{x^2}{2\sqrt{x^4+1}} \supset -\frac{y}{10} \subset +\frac{x}{10} \supset \frac{x}{10} \subset +\frac{y}{10} \supset$

Let $= z\cos((x+y)z) \subset +z\cos((x+y)z) \supset +(x+y)\cos((x+y)z)$. Compute $\div$.

$-((x+y)^2+2z^2)\sin((x+y)z)$ 0 $-z^2\sin((x+y)z) \subset -z^2\sin((x+y)z) \supset -(x+y)^2\sin((x+y)z)$ $(x+y)^2\sin((x+y)z)$ $yz\cos((x+y)z) - xz\sin((x+y)z)$

Let $= xz^2 \subset +yx^2 \supset +xyz$. Compute . $xz \subset +(2x-y)z \supset +2xy$ x^2+xy+z^2 $(2x+y) \subset +x \supset +2z$ $z^2 \subset +x^2 \supset +xy$ $(xy-x^2) \subset +(xy-z^2) \supset +(x^2-z^2)$

Let $\mathcal{C}$ be the curve defined by $2y = 1 - x^2$, $0 \le x \le 1$. Compute the value of the line integral $\int_{\mathcal{C}} x\, ds$.

$(2\sqrt{2}-1)/3$ $1/2$ $(\sqrt{3}-1)/2$ $1/4$ $7/3$

Let be the velocity field $x^2y \subset -xz^2 \supset +y^2$ and let $\mathcal{C}$ be the curve parameterized by $(t) = t \subset +t^2 \supset -2t$, $0 \le t \le 1$. Compute the flow integral $\int_{\mathcal{C}} d$. $-9/5$ $-7/3$ $-7/2$ $-9/4$ $-11/6$

Let $\mathcal{C}$ be a smooth curve from $(1,1,0)$ to $(0,0,1)$. Compute

$$\int_{\mathcal{C}} -yz\sin(xz)\,dx + \cos(xz)\,dy - xy\sin(xz)\,dz$$

-1 $-3\cos(1)$ $\cos(1)$ 2 0

Let $\mathcal{C}$ be the boundary of the region inside the unit circle in the first quadrant (oriented counterclockwise). Use Green's Theorem to evaluate $\int_{\mathcal{C}} x^2y\,dx - xy^2\,dy$.

$-\pi/8$ π $-\pi/3$ $\pi/4$ $-3\pi/4$

Determine which of the following formulas is true for an arbitrary function f or vector field . $\div = 0$ $\div f = 0$ $=$ $\div =$ $\div =$