Math 225: Calculus III Name:________________

Final Exam *December 19, 1991* Score:________________

Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this answer sheet. There are 25 multiple choice questions worth 6 points each.

Let $= 3 \subset - \supset$ and $\overline{=} \subset +2 \supset -$. Compute $\cdot$.

$1\ 0\ -2 \subset +3 \supset +7\ -$

Let $= 2 \subset +3 \supset -$ and $\overline{=} - \subset -2 \supset +4$. Compute $\times$.10ı- 7ȷ- 12ı+ ȷ- 4-12-12-4

Determine the parametric equations of the line through the point $(2, 3, 1)$ parallel to the vector $= 5 \subset -3 \supset +2$.

$x = 2 + 5t$

$y = 3 - 3t$

$z = 1 + 2t$

$x = 5 + 2t$

$y = -3 + 3t$

$z = 2 + t$

$x = 5t$

$y = -3t$

$z = 2t$

$x = 1 + t$

$y = 2 + t$

$z = t$

$x = -3 + 5t$

$y = 12 - 3t$

$z = 2t$

Determine the equation of the plane perpendicular to the vector $= 3 \subset +2 \supset +4$ passing through the point $(1, -1, 2)$.

$3x + 2y + 4z = 9$ $3x - 2y + 4z = 0$ $(x - 3) - (y - 2) + 2(z - 4) = 0$ $x - y + 2z = 9$ $x - y + 2z = 0$

Determine which of the following integrals gives the length of the curve parameterized by $(t) = \cos(t^2) \subset + \sin(t^2) \supset -t^2$, $0 \le t \le \sqrt{2\pi}$.

$\int_0^{\sqrt{2\pi}} 2t\sqrt{2}\, dt$ $\int_0^{\sqrt{2\pi}} \sqrt{1 + t^4}\, dt$ $\int_0^{2\pi} \sqrt{1 + 4t^2}\, dt$ $\int_0^{2\pi} 2t\sqrt{1 + t^2}\, dt$ $\int_0^{\pi} \sqrt{\cos(t^2) + \sin(t^2) + 4t^2}\, dt$

Find the parametric equations of the line tangent to the curve $(t) = \ln(t) \subset +t \supset +t^3$ at the point $(0, 1, 1)$.

$x = t$

$y = 1 + t$

$z = 1 + 3t$

$x = 1/t$

$y = 1$

$z = 3t^2$

$x = 1/t$

$y = 1 + t$

$z = 1 + 3t^2$

$x = 0$

$y = 1 + t$

$z = 1 + 3t^2$

$x = t$
$y = t$
$z = 3t$

Let $f(x, y) = x^3 \cos(y^2) + e^{-y^2} \sin(y)$. Compute f_{yx}. $-6x^2 y \sin(y^2)$ $3x^2 \cos(y^2) - 2x^3 y \sin(y^2)$ $3x^2 \cos(y^2) - 2ye^{-y^2} \sin(y) + e^{-y^2} \cos(y)$ $3x^2 \cos(y^2) - 2x^3 y \sin(y^2) - 2ye^{-y^2} \sin(y) + e^{-y^2} \cos(y)$ $-3x^2 \sin(y^2)$

Let $f(x, y) = y(x^2 + y^2)$ and suppose x and y are functions of u and v. If $x/du = 5$, $x/dv = 2$, $y/du = -3$ and $y/dv = 7$ when $(x, y) = (1, 2)$, compute f/du at that point. -19 99 33 53 -11

Calculate the derivative of the function $f(x, y, z) = x^3 y - z^2$ in the direction of the vector $2 \subset +4 \supset -4$ at the point $(1, -1, 3)$. 3.67 -7 22 10.33 -14

Determine the equation of the plane that is tangent to the graph of $f(x, y) = x^2 + y^2$ at the point $(1, 2, 5)$. $2x + 4y - z = 5$ $x + 2y = 5$ $2x + 4y - z = 0$ $x + 2y = 0$ $2x + 2y = z$

Determine which of the following statements describes the graph of $f(x, y) = 2x^3 - 3x^2 + y^2$ over the point $(1, 0)$

a local minimum a local maximum a saddle point not a critical point not continuous

Find the maximum of $f(x, y) = x^3 y$ subject to the constraint $2x^2 + y^2 = 1$. 0.1148 1.4159 0.8165 0 1.2236

Reverse the order of integration in the double integral $\int_{-2}^{2} \int_{x^2}^{4} f(x, y)\, dy\, dx$. $\int_0^4 \int_{-\sqrt{y}}^{\sqrt{y}} f(x, y)\, dx\, dy$ $\int_{x^2}^{4} \int_{-2}^{2} f(x, y)\, dx\, dy$ $\int_{-2}^{2} \int_{y^2}^{4} f(x, y)\, dx\, dy$ $\int_{-4}^{4} \int_0^{\sqrt{y}} f(x, y)\, dx\, dy$ $\int_0^4 \int_{-\sqrt{y}}^{2} f(x, y)\, dx\, dy$

Calculate the area of the region bounded above by the circle of radius π and below by the x-axis and the spiral $r = \theta$ in polar coordinates.

$\frac{1}{3}\pi^3$ $2\pi - \frac{1}{6}\pi^3$ $\pi^2 - \frac{1}{6}\pi^3$ $\frac{1}{2}\pi^2$ $\frac{1}{2}\pi^2 - \frac{1}{6}\pi^3$

Which of the following integrals gives the volume of the solid in the first octant bounded by the plane $x + y = 4$ and the cylinder $y^2 + 4z^2 = 16$.

$\int_0^4 \int_0^{4-y} \int_0^{\frac{1}{2}\sqrt{16-y^2}} 1\, dz\, dx\, dy$ $\int_{-4}^{4} \int_0^{4-y} \int_0^{\sqrt{16-y^2}} 1\, dz\, dx\, dy$ $\int_0^2 \int_0^{4-x} \int_0^{\frac{1}{2}\sqrt{16-y^2}} 1\, dz\, dx\, dy$ $\int_0^4 \int_0^{x+y} \int_0^{\frac{1}{2}\sqrt{16-y^2}} 1\, dz\, dy\, dx$ $\int_0^{16} \int_0^{x+y} \int_0^{y^2+4z^2} 1\, dz\, dy\, dx$

Let D be the part of the cylinder $x^2 + y^2 = 4$ below the plane $x + z = 3$ and above the xy-plane. The volume of D is 12π. Compute the z-coordinate of the centroid of D.

1.67 1.33 2.00 2.33 2.67

Determine which of the following integrals gives the volume of the part of the solid sphere $x^2+y^2+z^2 \leq 9$ cut out by the cone $z = \sqrt{x^2+y^2}$.

$\int_0^{2\pi} \int_0^{\pi/4} \int_0^3 \rho^2 \sin(\phi)\, d\rho\, d\phi\, d\theta$ $\int_0^{2\pi} \int_0^{\pi/3} \int_0^3 \rho^2 \sin(\phi)\, d\rho\, d\phi\, d\theta$ $\int_0^{2\pi} \int_0^{\pi/6} \int_0^3 1\, d\rho\, d\phi\, d\theta$ $\int_0^{2\pi} \int_0^{2\pi/3} \int_0^9 \rho \sin(\theta)\, d\rho\, d\phi\, d\theta$ $\int_0^{2\pi} \int_0^{\pi/2} \int_0^9 \rho \cos(\phi)\, d\rho\, d\phi\, d\theta$

Use the substitution $x = u^2+v^2$, $y = u^2 - v^2$ to transform the integral $\int_0^1 \int_y^{y+8} e^{x^2-y^2}\, dx\, dy$. (Assume $u \geq 0$ and $v \geq 0$.)

$\int_0^2 \int_v^{\sqrt{1+v^2}} 8uve^{4u^2v^2}\, du\, dv$ $\int_0^1 \int_v^{v+8} e^{4u^2v^2}\, du\, dv$ $\int_0^1 \int_{-v}^{v} 8e^{4u^2v^2}\, du\, dv$ $\int_0^2 \int_v^2 \sqrt{uv}e^{u^2-v^2}\, du\, dv$ $\int_0^2 \int_v^{v^2} \sqrt{1+v^2}e^{u^2+v^2}\, du\, dv$

Compute the line integral $\int_C (z^2+xy)\, dx + (y^2-xz)\, dy + dz$ where C is the curve parameterized by $(t) = t^3 \subset -t \supset +t^2$, $0 \leq t \leq 1$.

5/6 11/15 5/3 0 13/12

Use the Fundamental Theorem of Line Integrals to calculate $\int_C 2xy\, dx + (x^2 - 3y^2 z)\, dy + (1-y^3)\, dz$ where C is the curve parameterized by $(t) = e^{-t^2}\cos(t) \subset +\sin(t) \supset +t$, $0 \leq t \leq \pi/2$.

0 $\pi/2$ π $\pi/2 - 1$ $\pi - 1$

Let C be the boundary of the triangle with vertices $(0,0)$, $(2,0)$, and $(0,1)$ oriented counter-clockwise. Use Green's Theorem to express the line integral $\int_C (e^{x^2}+y^2)\, dx + (e^{y^2}+x^2)\, dy$ as a double integral.

$2\int_0^2 \int_0^{1-\frac{1}{2}x} (x-y)\, dy\, dx$ $\int_0^2 \int_0^1 (x+y)\, dx\, dy$ $2\int_0^2 \int_0^{2x+1} (x-y)\, dy\, dx$ $2\int_0^1 \int_0^{1-\frac{1}{2}y} (ye^{y^2} - xe^{x^2})\, dx\, dy$ $\int_0^1 \int_0^{2y+1} (ye^{y^2} + xe^{x^2})\, dx\, dy$

Calculate the surface area of the part of the paraboloid $x^2+y^2+z = 16$ above the xy-plane.

$\frac{\pi}{6}[(65)^{3/2} - 1]$ $16\sqrt{16-\pi^2}$ $\frac{\pi}{3}[(17)^{3/2}-1]$ $\frac{4\pi}{3}$ $4\sqrt{1+4\pi^2}$

Let Σ be the portion of the sphere of radius 3 in the first octant and let be the unit upward normal vector to Σ. Compute the flux integral $_\Sigma (z \subset +x) \cdot\, d\sigma$.

18 9π 0 3π 6

Let C be the boundary of the triangle cut out from the plane $x + y + z = 1$ by the first octant. Use Stokes' Theorem to calculate the line integral $\int_C (y \subset +z \supset +x) \cdot d$

$-\frac{3}{2}$ 3 $-\frac{1}{2}$ 0 2

Let D be the solid rectangular box defined by $-1 \leq x \leq 1$, $-1 \leq y \leq 1$, and $0 \leq z \leq 2$. Let Σ be the boundary of this box and let be its outward unit normal vector. Use the Divergence Theorem to express the flux integral $_{\Sigma}(x^2 \subset +y^2 \supset +z^2) \cdot d\sigma$ as a triple integral.

$2\int_0^2 \int_{-1}^1 \int_{-1}^1 (x+y+z)\, dx\, dy\, dz$ $\int_0^2 \int_{-1}^1 \int_{-1}^1 0\, dx\, dy\, dz$ $\int_0^2 \int_{-1}^1 \int_{-1}^1 (x^2+y^2+z^2)\, dx\, dy\, dz$ $\int_0^2 \int_{-1}^1 \int_{-1}^1 \frac{x \subset +y \supset +z}{z}\, dx\, dy\, dz$ $2\int_0^2 \int_{-1}^1 \int_{-1}^1 (x \subset +y \supset +z)\, dx\, dy\, dz$