Math 225: Calculus III Name:______________________________

Final Exam *May 2, 1994* Section:______________________________

Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this answer sheet. There are 25 multiple choice questions worth 5 points each. You start with 25 points.

Find the area of the parallelogram with vertices $(0,0,0)$, $(1,2,-1)$, $(0,1,1)$, and $(1,3,0)$.

3.32 2.87 3.94 2.60 2.35

Find the parametric equations of the line that passes through the point $(1,0,-1)$ and is perpendicular to the plane $2x-y+3z=7$.

$x = 1+2t$
$y = -t$
$z = -1+3t$

$2(x-1)-y+3(z+1)=7$ $x = 2+t$, $y = -1$, $z = 3-t$ $x = 2t$, $y = -t$, $z = 3t$ $x = t$, $y = 0$, $z = -t$

Find the equation of the plane through the point $(3,1,5)$ perpendicular to the y-axis.

$y=1$ $x+z=8$ $3x+y+5z=9$ $x+y+z=9$ $3(x-3)+(y-1)+5(z-5)=0$

A particle's position is given by $(t)=\cos(\pi t)\subset+\sin(\pi t)\supset+t^2$. Find its speed at $t=2$.

$\sqrt{16+\pi^2}$ $-\pi\subset+\pi\supset+4$ $\sqrt{17}$ 2π $4\supset$

Suppose a particle's acceleration is $(t)=e^t\subset+e^{-t}\supset+$. Find the particle's position at time $t=1$ if it is initially at rest at the origin.

$(e-2)\subset+e^{-1}\supset+\frac{1}{2}$ $e\subset+e^{-1}\supset+\frac{1}{2}$ $(e-2)\subset+(e^{-1}+2)\supset+$ $e\subset+(e^{-1}+2)\supset+\frac{1}{2}$ $(e-1)\subset+(e^{-1}+1)\supset+$

Let $(t)=2t\subset+3t^2\supset-t^3$. Find the unit tangent vector, (t), at $t=1$. $\frac{2}{7}\subset+\frac{6}{7}\supset-\frac{3}{7}$ $2\subset+2\supset-3$ $2\subset+6t\supset-3t^2$ $\frac{2}{\sqrt{4+4t^2+9t^4}}\subset+\frac{2t}{\sqrt{4+4t^2+9t^4}}\supset-\frac{3t^2}{\sqrt{4+4t^2+9t^4}}$ $\frac{2}{\sqrt{17}}\subset+\frac{2}{\sqrt{17}}\supset-\frac{3}{\sqrt{17}}$

Compute $\lim_{(x,y)\to(0,0)}\frac{x^3+y^3}{x^2+y^2}$. 0 1 2 $\frac{1}{2}$ *does not exist*

Let $f(x,y)=xe^{xy}$. Compute $f_{xy}(2,1)$. $8e^2$ $3e^2$ $2e^2$ $4e^2$ $6e^2$

The position and velocity of a smoothly moving particle at time $t=1$ are $(1)=3\subset+4\supset$ and $\check{}(1)=5\subset-5\supset$, respectively. Determine how fast the distance of the particle to the origin is changing at time $t=1$.

-1 -5 $5\sqrt{2}$ $-\sqrt{2}/2$ 35

Find the direction in which the function $f(x,y)=\sin(x^2+xy)$ is increasing most rapidly at $(0,1)$.

$\subset$ $\supset$ $\subset+\supset$ $\subset-\supset$ $2\subset+\supset$

Find the equation of the plane tangent to the graph of $f(x,y)=x^2y-y^3$ at the point $(2,1)$.

$4x+y-z=6$ $4x+y-z=0$ $2xy+x^2-3y^2=5$ $4x+y=9$ $4x+y=0$

Find all of the critical points of the function $f(x,y)=x^2y-6xy+8y-x^2+6x-8$.

$(2,1)$, $(4,1)$ $(2,1)$, $(3,1)$, $(4,1)$ $(2,1)$, $(2,-1)$, $(3,1)$, $(3,-1)$, $(4,1)$, $(4,-1)$ $(2,0)$, $(2,1)$, $(4,0)$, $(4,1)$, $(3,0)$, $(3,1)$ $(3,1)$

Let $f(x,y)=x^3y-3x^2y$. determine which of the following statements is true.

f has a saddle point at $(3,0)$. f has a local minimum at $(3,0)$. f has a local maximum at $(3,0)$. f is not continuous at $(3,0)$. $(3,0)$ is not a critical point of f.

Determine which of the following sysytems of equations must be solved to find the extrema of the function $f(x,y)=x^2y-y^3$ subject to the constraint $x^4+y^4=1$ using the method of Lagrange multipliers.

$$\begin{aligned} xy &= 2\lambda x^3 \\ x^2-3y^2 &= 4\lambda y^3 \\ x^4+y^4 &= 1 \end{aligned}$$

$$\begin{aligned} 2xy &= \lambda x^4 \\ x^2-3y^2 &= \lambda y^4 \\ x^4+y^4 &= 1 \end{aligned}$$

$$\begin{aligned} xy &= 0 \\ x^2-3y^2 &= 0 \\ x^3+y^3 &= 0 \end{aligned}$$

$$\begin{aligned} xy &= 0 \\ x^2 - 3y^2 &= 0 \\ x^4 + y^4 &= 1 \\ xy &= 4\lambda x^3 \\ x^2 - 3y^2 &= 4\lambda y^3 \\ x^4 + y^4 &= 0 \end{aligned}$$

Find the area enclosed by one leaf of the rose $r = 6\sin(2\theta)$.

$9\pi/2$ 18π 9π 4π 6π

Find the centroid of the triangle in the xy-plane with vertices $(0,0)$, $(2,1)$, $(2,0)$.

$(\frac{4}{3}, \frac{1}{3})$ $(\frac{3}{4}, \frac{1}{4})$ $(\frac{5}{4}, \frac{1}{4})$ $(\frac{5}{4}, \frac{1}{3})$ $(\frac{4}{3}, \frac{1}{4})$

Compute the volume of the portion of the solid region bewteen the spheres $\rho = 1$ and $\rho = 2$ that lies inside the upper nappe of the cone $x^2 + y^2 = z^2$.

$7\pi(2-\sqrt{2})/3$ $5\pi(\sqrt{2}-1)/3$ $14\pi/3$ $5\sqrt{2}\pi/3$ 7π

Find the average value of the height function $f(x,y,z) = z$ in the solid region D inside the cylinder $x^2 + y^2 = 1$ between the planes $x + y + z = 4$ and $z = 0$. The volume of D is 4π.

2.06 2.84 2.57 3.21 2.28

Compute the Jacobian $\frac{\partial(x,y)}{\partial(u,v)}$ of the change of coordinates $u = 5x + 2y$ and $v = y - 2x$.

$\frac{1}{9}$ 9 5 $\frac{1}{5}$ -12

Let $\mathcal{C}$ be the curve defined by $(t) = t^3 \subset +t^2 \supset$, $0 \le t \le 1$. Compute $\int_{\mathcal{C}} (9x + 2\sqrt{y})\, ds$.

$\frac{13^{3/2}}{6}$ $\frac{17^{3/2}}{2}$ $\frac{15^{3/2}}{3}$ $\frac{11^{3/2}}{2}$ $\frac{9}{2}$

Use the Fundamental Theorem of Line Integrals to compute $\int_{\mathcal{C}} (2xy+1)\, dx + (x^2+3y^2-z)\, dy + (6z-y)\, dz$. where $\mathcal{C}$ is a smooth curve from $(0,0,0)$ to $(1,2,3)$.

32 16 8 64 4

Compute the surface area of the portion of the paraboloid $x^2 + y^2 + z = 9$ above the plane $z = 5$.

$\frac{\pi}{6}(17^{3/2} - 1)$ $\frac{\pi}{2}(11^{3/2} - 1)$ $\frac{\pi}{12}(13^{3/2} - 1)$ $\frac{\pi}{3}(15^{3/2} - 1)$ $\frac{13\pi}{3}$

Let Σ be the part of the sphere $x^2 + y^2 + z^2 = 4$ above the triangle in the xy-plane with vertices $(0,0,0)$, $(1,1,0)$, and $(1,0,0)$. Let $= y \subset -x \supset +2x$ and let be the upward unit normal vector to Σ. Compute the flux integral ${}_{\Sigma}\, d\sigma$.

2/3 3/4 4/5 1/2 7/8

Let $\mathcal{C}$ be the intersection of the plane $y+z = 1$ with the cylinder $x^2+y^2 = 4$, oriented counterclockwise. Let $= x^2 \subset +xy \supset +z^2$. Use Stokes' Theorem to compute $\int_{\mathcal{C}} d$.

0 2π $3\pi/4$ $5\pi/16$ $\pi/2$

Let Σ be the unit sphere with outward unit normal vector . Let $= x^3 \subset +y^3 \supset +z^3$. Use the Divergence Theorem to compute the flux integral ${}_{\Sigma}\, d\sigma$.

$12\pi/5$ $16\pi/3$ 6π $9\pi/2$ 2π