Math 225: Calculus III Name:________________________

Exam II *October 29, 1992* Section:________________________

Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this answer sheet. There are 15 multiple choice questions worth 6 points each. You start with 10 points.

Let $f(x,y,z) = \frac{\ln(xy^2)}{z^3}$. Compute $f_{zx}(1,2,3)$. $-\frac{1}{27}$ $-\frac{2}{\ln(2)81}$ 0 $-\frac{2}{\ln(2)27}$ $-\frac{1}{3}$

Compute the limit $\lim_{(x,y)\to(0,0)} \frac{xy^3}{x^4+y^4}$. *does not exist* $\frac{3}{8}$ $\frac{1}{8}$ ∞ 0

Calculate the gradient of the function $f(x,y,z) = x^2y - z^2x + yz$. $(2xy - z^2) \subset +(x^2+z) \supset +(y-2xz)$ $2xy - z^2 + x^2 + z + y - 2xz$ $2xy - z^2 + y + z$ $x^2y \subset -z^2x \supset +yz$ $2xy \subset -z^2 \supset +(y+z)$

Find the direction in which the function $f(x,y) = xy - x/y$ is increasing most rapidly at the point $(5,3)$. $\frac{8}{3} \subset +\frac{50}{9} \supset$ $3 \subset -\frac{5}{9} \supset$ $\frac{74}{9}$ $\frac{24}{5} \subset +\frac{78}{25} \supset$ $\frac{40}{3}$

Calculate the derivative of $f(x,y,z) = xy - z^2$ in the direction of $\subset - \supset +2$ at the point $(3,4,1)$. $-\frac{3}{\sqrt{6}}$ $4 \subset +3 \supset -2$ -3 1 $\frac{4}{\sqrt{6}} \subset -\frac{3}{\sqrt{6}} \supset -\frac{4}{\sqrt{6}}$

Determine the critical points of the function $g(u,v) = u^2v - 27v^3 - 2u$. $(3,\frac{1}{3}),(-3,-\frac{1}{3})$ $(3,\frac{1}{3}),(-3,\frac{1}{3}),(3,-\frac{1}{3}),(-3,-\frac{1}{3})$ $(9,\frac{1}{9}),(-9,-\frac{1}{9}),(0,\frac{1}{9}),(0,-\frac{1}{9})$ $(0,0),(1,1),(-1,-1)$ $(1,1),(9,1),(-1,-1),(-9,-1)$

Which of the following statements is true about the function $f(x,y) = e^{x^2-3xy+y^2}$. $f(x,y)$ has a critical point at $(1,-1)$ $f(x,y)$ has a local maximum at $(0,0)$ $f(x,y)$ has a local minimum at $(0,0)$ $f(x,y)$ has a saddle point at $(0,0)$ *none of the above*

Find the minimum value of the function $f(x,y) = x^2 - 2xy + y - x$ in the square region $0 \le x \le 2$, $-1 \le y \le 1$. $-\frac{5}{4}$ $-\frac{1}{4}$ -2 -1 $-\frac{1}{2}$

Determine the maximum value of the function $f(x,y) = x - 2y$ subject to the constraint $x^2 + 4y^2 = 4$.
2.828 1.000 1.436 2.000 3.936

Which of the following sets of equations must be solved to find the extreme values of the function $f(x,y) = x^4y + x^2y + xy^3$ subject to the constraint $x^4 + 4xy + y^4 = 16$

$4x^3y + 2xy + y^3 = \lambda(4x^3 + 4y)$
$x^4 + x^2 + 3xy^2 = \lambda(4x + 4y^3)$
$x^4 + 4xy + y^4 = 16$

$4x^3y + 2xy + y^3 = \lambda(4x^3 + 4y)$
$x^4 + x^2 + 3xy^2 = \lambda(4x + 4y^3)$
$4x^3 + 4y + 4y^3 = 0$

$4x^3 + 2xy + y^3 = \lambda(4x^3 + 4y^3)$
$x^2 + 3xy^2 = \lambda(4x + 4y^3)$

$4x^3 + 2xy = \lambda(4x^3 + 4y)$
$x^2 + 3xy^2 = \lambda(4x + 4y^3)$
$x^4 + 4xy + y^4 = 16$

$4x^3 + x^4 + 2xy + x^2 + 3y^2 = \lambda(4x^3 + 4y)$
$x^4 + 4x^3y + x^2 + 3xy^2 = \lambda(4x + 4y^3)$
$x^3 + x + y + y^3 = 4$

Find the equation of the plane tangent to the graph of $g(x,y) = x^3y - y^2 + x$ at the point $(1,1,1)$. $4x - y - z = 2$ $4x - y = 3$ $x + y + z = 3$ $x + y = 2$ $(1 + 3x^2y) + (x^3 - 2y) + z = 4$

Let $z = x^2\cos(y + x)$. Suppose $x = g(u,v)$, $y = h(u,v)$ and at the point $(u,v) = (0,0)$, $g(0,0) = \pi$, $h(0,0) = 0$, and:

$$x/du = 3, \quad x/dv = -1, \quad y/du = -2, \quad y/dv = -2$$

Compute z/du at $(u,v) = (0,0)$.
-6π 0 4π -2π 1

A certain type of salt crystal is being grown in a chemistry lab. The crystal is the shape of a rectangular box. After 1 hour, the crystal is observed to be 1 mm wide, 2 mm deep, and 2.5 mm high with the width, depth, and height increasing at a rate of 0.2 mm, 0.4 mm, and 0.5 mm per hour, respectively. How fast is the volume of the crystal changing at that moment. 3.0 mm^3 per hour 2.25 mm^3 per hour 1.8 mm^3 per hour 1.1 mm^3 per hour 2.75 mm^3 per hour

Determine which of the following plots represents the level curves of the function $f(x, y) = 2x^2 + 4y^2$.

Which of the following plots represents the graph of the function $f(x, y) = 2y^3 + 3x^2 - 6xy$