Math 225: Calculus III Name:__________________________

Exam III *December 1, 1992* Section:__________________________

Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this answer sheet. There are 15 multiple choice questions worth 6 points each. You start with 10 points.

Compute $\int_1^e \int_0^{1/y} \ln(y)\, dx\, dy$.

$\frac{1}{2}$ $\ln(2)$ 0 1 2

Reverse the order of integration in the integral $\int_{-\sqrt{2}}^{\sqrt{2}} \int_1^{3-x^2} f(x,y)\, dy\, dx$.

$\int_1^3 \int_{-\sqrt{3-y}}^{\sqrt{3-y}} f(x,y)\, dx\, dy$ $\int_1^{3-x^2} \int_{-\sqrt{2}}^{\sqrt{2}} f(x,y)\, dx\, dy$ $\int_{-\sqrt{2}}^{\sqrt{2}} \int_1^{3-y^2} f(x,y)\, dx\, dy.$ $\int_0^3 \int_0^{3-y^2} f(x,y)\, dx\, dy$ $\int_{-2}^{2} \int_{-\sqrt{3-y}}^{\sqrt{3-y}} f(x,y)\, dx\, dy$

Find the y-coordinate of the centroid of the plane region bounded by $y = 2x$ and $y = x^2$.

1.6 2.4 2.2 1.8 2.6

Calculate the area between the spirals $r = \theta$ and $r = 2\theta$ above the x-axis.

$\pi^3/2$ $\pi^2/2$ π^2 $\pi^3/6$ 2π

Evaluate ${}_R e^{-x^2-y^2}\, dA$ where R is the region $x^2 + y^2 \leq 1$.

$\pi(1-e^{-1})$ $2\pi\, e^{-1}$ $(1-e^{-1})/2$ $\pi(e-1)^{-1}$ π

Evaluate the iterated integral $\int_0^{\sqrt{\pi/2}} \int_0^y \int_0^{xy} \sin(x^2)\, dz\, dx\, dy$.

$(\pi-2)/8$ $\pi/4$ $\sqrt{2\pi}$ $(\pi-1)/4$ $\sqrt{\pi}$

Determine which of the following sets of inequalities describes the solid region bounded above by the sphere $x^2 + y^2 + z^2 = 80$ and below by the paraboloid $2z = x^2 + y^2$.

$\frac{1}{2}r^2 \leq z \leq \sqrt{80 - r^2}$
$0 \leq r \leq 4$
$0 \leq \theta \leq 2\pi$

$0 \leq \rho \leq \sqrt{80}$
$0 \leq \phi \leq \pi/6$
$0 \leq \theta \leq 2\pi$

$0 \leq z \leq \sqrt{80 - r^2}$
$0 \leq r \leq 8$
$0 \leq \theta \leq \pi$

$\frac{1}{2}r \leq \rho \leq \sqrt{80}$
$0 \leq \phi \leq \pi/3$
$0 \leq \theta \leq \pi$

$\frac{1}{2}r^2 \leq z \leq \sqrt{80}$
$0 \leq r \leq \sqrt{40}$
$0 \leq \theta \leq 2\pi$

Let D be the tetrahedron bounded by the planes $2x + y + z = 10$, $x - 2y = 0$, $x = 3$ and $z = 0$. Determine which of the following integrals gives ${}_D f(x, y, z)\, dV$.

$\int_3^4 \int_{x/2}^{10-2x} \int_0^{10-y-2x} f(x, y, z)\, dz\, dy\, dx$ $\int_0^3 \int_0^{10-2x} \int_0^{x/2} f(x, y, z)\, dz\, dy\, dx$ $\int_0^3 \int_{2y}^{(10-y)/2} \int_0^{10-y-2x} f(x, y, z)\, dz\, dx\, dy$ $\int_3^5 \int_{x/2}^{10-2x} \int_{2y}^{10-y-2x} f(x, y, z)\, dz\, dy\, dx$ $\int_0^4 \int_0^{10-2x} \int_0^{10-y} f(x, y, z)\, dz\, dy\, dx$

Let D be the region cut from the solid sphere of radius a by the veritcal planes $y = x$ and $y = 0$ in the first octant. Suppose the density of D is given by $\delta(x, y, z) = bz$. Compute the total mass of D (a and b are constants).

$\frac{\pi a^4 b}{32}$ $\frac{\pi a^3 b}{8}$ $\frac{\pi a^3 b}{4}$ $\frac{\pi a^2 b^2}{16}$ $\frac{\pi a^3 b^2}{2}$

Let R be the region bounded by the lines $y = x$, $y = 4x$, $y = 1/x$, and $y = 4/x$. Use the transformation $x = uv$, $y = u/v$ to rewrite the integral ${}_R \sqrt{xy}\, dA$ as an integral over a region in the uv-plane.

$\int_{1/2}^{1}\int_{1}^{2} \frac{2u^2}{v}\,du\,dv$ $\int_{1}^{2}\int_{1}^{2} u\,du\,dv$ $\int_{1/4}^{1}\int_{1}^{4} -\frac{2u}{v}\,du\,dv$ $\int_{0}^{1}\int_{1}^{4} \frac{u^2}{v}\,du\,dv$ $\int_{1}^{4}\int_{1}^{4} -\frac{u}{2v}\,du\,dv$

Let $(x, y, z) = x^3 z \subset +(y^2 + z^2) \supset +z^3 x$. Compute $\div$.

$3x^2 z + 2y + 3z^2 x$ $3x^2 z \subset +2y \supset +3z^2 x$ $(3x^2 z + z^3) \subset +2y \supset +(x^3 + 3z^2 x)$ $3x^2 z + z^3 + 2y + x^3 + 3z^2 x$ $-2z \subset +(x^3 - z^3) \supset$

Let $= (x^3 y + z^2) \subset +(yz - x) \supset +xyz^2$. Compute .

$(xz^2 - y) \subset +z(2 - yz) \supset -(x^3 + 1)$ $3x^2 y + z + 2xyz$ $z^2(x - y) - x^3 + 2z - y + 1$ $(yz^2 - 1) \subset -z(1 + xz) \supset -(z + x^3)$ $\mathbf{0}$

Let be the directed line segment from the point $(1, 2)$ to the point $(5, 3)$. Compute the value of line integral $\int_x y\,ds$.

$\frac{47\sqrt{17}}{6}$ $\frac{221}{2}$ 30 $\frac{23\sqrt{13}}{2}$ $\frac{331}{3}$

Compute the flow integral $\int_{\cdot} d$ of the velocity field $(x, y, z) = y \subset -x \supset +$ along the helix $(t) = \cos(\pi t) \subset + \sin(\pi t) \supset +t$, $0 \le t \le 4$.

$4(1 - \pi)$ $(\pi - 1)/4$ 2π $\pi/4$ 0

Use the Fundamental Theorem of Line Integrals to evaluate $\int_y (z + 1)\,dx + (2y + x(z + 1))\,dy + xy\,dz$ where is the curve $(t) = te^{(t-1)} \subset +2te^{(t-1)^2} \supset +3te^{(t-1)^3}$, $0 \le t \le 1$.

12 10 8 6 4