Math 225: Calculus III Name:________________________

Final Exam *December 14, 1992* Section:________________________

Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this answer sheet. There are 25 multiple choice questions worth 6 points each.

Use the Divergence Theorem to calculate the flux integral $_\Sigma \cdot dS$ where $= yz^2 \subset +zx^2 \supset +xy^2$, Σ is the sphere $x^2+y^2+z^2=9$, and is the outward unit normal vector to Σ.

0 2 3 36π $4\pi^2$

Use Stokes' Theorem to calculate the flow integral $\int_{\cdot} d$ where is the boundary of the rectangle with vertices $(0,0,1)$, $(1,0,1)$, $(1,1,0)$, $(0,1,0)$ in the plane $y+z=1$ and $=(x+y)\subset +(y+z)\supset +(x+z)$.

-2 -1 0 1 2

Let $= x \subset +3y \supset -2xz$ and let Σ be the portion of the cylinder $y^2+z^2=1$ above the xy-plane between $x=0$ and $x=1$. Determine which of the following integrals gives the value of the flux integral $_\Sigma \cdot d\sigma$ where is the upward unit normal vector to Σ.

$\int_{-1}^{1}\int_0^1 3y^2(1-y^2)^{-1/2}-2x(1-y^2)^{1/2}\,dx\,dy$ $\int_0^{2\pi}\int_0^1 3r^3(1-r^2)^{1/2}r\,dr\,d\theta$ $\int_{-1}^{1}\int_0^1 3x^2(1-y^2)^{1/2}-2y(1-y^2)^{3/2}\,dx\,dy$ $\int_0^{2\pi}\int_0^1 3r^2\cos(\theta)(1-r^2)^{3/2}\,dr\,d\theta$ $\int_0^1\int_0^1 3y(1-y^2)^{1/2}-2x^2(1-y^2)^{-1/2}\,dx\,dy$

Calculate the surface area of the portion of the paraboloid $x^2+y+z^2=4$ in the first octant. 9.04 10.21 13.37 8.62 5.79

Let be the curve parameterized by $(t)=(t^3-1)\subset +t^2\supset +t$, $0\le t\le 2$. Compute the flow integral $\int_{(} y-x)\,dx+(z-y)\,dy+2xz\,dz$.

1.33 4.5 6.00 8.67 2.50

Let be the directed line segment from $(-1,2,-1)$ to $(5,0,2)$. Calculate the line integral $\int_{(} x+y+z)\,ds$. 24.5 $\sqrt{15}/2$ $\sqrt{31}/2$ 48.5 72.5

Let $(x,y,z)=\frac{x}{y}\subset +\frac{y}{z}\supset +\frac{z}{x}$. Compute $\div$. $\frac{xz+xy+yz}{xyz}$ $\frac{y}{z^2}\subset +\frac{z}{x^2}\supset +\frac{x}{y^2}$ $\frac{x^2y+y^2z+xz^2}{x^2y^2z^2}$ $\frac{1}{y}\subset +\frac{1}{z}\supset +\frac{1}{x}$ $\frac{-x}{y^2}-\frac{y}{z^2}-\frac{z}{x^2}$

Determine which of the following vector fields is depicted below.

$\frac{(-y\subset+x\supset)}{\sqrt{x^2+y^2}}$ $\frac{x}{5}\subset+\frac{y}{5}j$ $\frac{y}{5}\subset-\frac{x}{5}\supset$ $\frac{x\subset-y\supset}{\sqrt{x^2+y^2}}$ $x^2\subset+y^2\supset$

Let R be the region bounded by the lines $x+2y=3$, $x+2y=5$, $2x-y=1$, and $2x-y=2$. Use a substitution to calculate the integral $_R(x-y)\,dA$.

1/25 1/16 1/5 1/4 1/12

Which of the following integrals gives the volume of the portion of the solid sphere $x^2+y^2+z^2\leq 4$ above the xy-plane and below the cone $x^2+y^2=3z^2$.

$\int_0^{2\pi}\int_{\pi/3}^{\pi/2}\int_0^2\rho^2\sin(\phi)\,d\rho\,d\phi\,d\theta$ $\int_0^{2\pi}\int_0^{\pi/2}\int_0^2\rho^2\sin(\phi)\,d\rho\,d\phi\,d\theta$ $\int_0^{2\pi}\int_0^1\int_0^{\sqrt{4-r^2}}r\,dz\,dr\,d\theta$ $\int_0^{\pi/2}\int_0^1\int_0^{\sqrt{4-r^2}}r\,dz\,dr\,d\theta$ $\int_0^{\pi/2}\int_{\pi/6}^{\pi}\int_0^2\rho^2\sin(\phi)\,d\rho\,d\phi\,d\theta$

A mound of earth has the shape of an inverted paraboloid $z=1-x^2-y^2$. The density of material in the mound is given by $\delta(x,y,z)=1-z$. Compute the total mass of the mound.

$\pi/3$ $\pi/2$ $\pi/4$ $\pi/9$ $\pi/6$

Find the average value of the function $f(x, y) = x^2$ over the triangular region bounded by the lines $y = x$, $y = 0$ and $x = 2$.

2 4 6 1 3

Compute the area of the region inside the cardioid $r = 3 + 2\cos(\theta)$.

11π 9π 6π $\frac{16}{3}\pi$ $\frac{32}{3}\pi$

Compute $\int_0^1 \int_x^{x^2} \int_0^{y-x} (x + y)\, dz\, dy\, dx$ 1/70 1/35 1/15 1/105 1/210

Rewrite the integral $\int_1^4 \int_{4/x^2}^{(21-5x)/4} f(x, y)\, dy\, dx$ by reversing the order of integration.

$\int_{1/4}^4 \int_{2/\sqrt{y}}^{(21-4y)/5} f(x, y)\, dx\, dy$ $\int_{4/x^2}^{(21-5x)/4} \int_1^4 f(x, y)\, dx\, dy$ $\int_1^4 \int_{4/y^2}^{(21-4y)/5} f(x, y)\, dx\, dy$ $\int_0^2 \int_{2/y}^{(21-5y)/4} f(x, y)\, dx\, dy$ $\int_0^2 \int_{2/\sqrt{y}}^{(21-5y)/4} f(x, y)\, dx\, dy$

Find the minimum value of the function $f(x, y) = x - y$ on the ellipse $4x^2 + y^2 = 1$.

-1.12 -2.23 -1.00 -0.50 -1.73

Which of the following statements applies to the function $f(x, y) = x^3 - 3x^2 + 3y^2$.

$(0, 0)$ is *not* a critical point of f f has a saddle point at $(2, 0)$ f has a local maximum at $(0, 0)$ f has a local minimum at $(2, 0)$ *none of the above*

Find all of the critical points of the function $f(x, y) = x^2(2 - 3x^2) + 12xy(x^2 - 1)$.

$(1, 1/3)$, $(-1, -1/3)$, $(0, 0)$ $(1, 1/3)$, $(1, -1/3)$, $(-1, 1/3)$, $(-1, -1/3)$ $(1, -1/3)$, $(-1, 1/3)$, $(0, 0)$ $(1, 1/3)$, $(-1, 1/3)$, $(-1, -1/3)$, $(0, 0)$ $(1, 1/3)$, $(1, -1/3)$, $(0, 0)$

Find the maximum value of $f(x, y) = e^{xy-x-2y}$ on the region $x + y \le 7$, $x \ge 0$, $y \ge 0$.

e^2 e e^{-1} 1 e^4

Determine the equation of the plane tangent to the surface $x^2 - yz = 4$ at the point $(2, 0, 0)$.

$x = 2$ $2x - y - z = 0$ $2x - y - z = 4$ $4x - y = 0$ $4x - y = 8$

Find the direction in which the function $f(x, y, z) = x^3y - y^2z + z^4$ increases most rapidly at the point $(1, 2, -1)$.

$6 \subset +5 \supset -8$ $3 \subset -2 \supset +4$ $3 \subset - \supset +3$ $6 \subset +4 \supset -4$

Let $f(x, y) = x^3y^2$. If $x = r\cos(\theta)$ and $y = r\sin(\theta)$, determine which of the expressions below gives $f/d\theta$.

$rx^2y(2x\cos(\theta) - 3y\sin(\theta))$ $rx^2y(3y\cos(\theta) + 2x\sin(\theta))$ $-3r^5\cos^2(\theta)\sin^3(\theta)$ $2r^5\cos^4(\theta)\sin(\theta)$ $6rx^2y(\sin(\theta) - \cos(\theta)$

Determine the equation of the line perpendicular to the plane $2x - 3y + 4z = 5$ through the point $(1, 0, 0)$.

$x = 1 + 2t, \quad y = -3t, \quad z = 4t$ $x = 2 + t, \quad y = -3 + t, \quad z = 4 + t$ $x = 1 + t, \quad y = t, \quad z = t$ $x = 1 + t/2, \quad y = -t/3, \quad z = t/4$ $x = 5 - 4t, \quad y = -3t + 3, \quad z = 4t - 4$

Compute the angle in radians between the lines $x = 2 - t$, $y = 3 - 2t$, $z = 4 - 3t$, and $x = 1 + t$, $y = 4 - t$, $z = 4t$.

2.34 0.79 1.05 1.67 2.09

A charged particle with mass $m = 2 \times 10^{-2}$ moves under the influence of a changing electrical force, $(t) = (1 - t) \subset +t^2 \supset$ (t in seconds). If the particle is initially at rest at the origin, determine the particle's position after 2 seconds.

$(100 \subset +200 \supset)/3$ $(2 \subset +4 \supset)/3$ $(50 \subset +100 \supset)/3$ $(\subset +8 \supset)/6$ $(50 \subset +400 \supset)/6$