Math 225: Calculus III Name:________________________________

Exam I *February 3, 1994* Section:________________________________

Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this answer sheet. There are 15 multiple choice questions worth 6 points each. You start with 10 points.

Particle a has position vector $(t) = t^2 \subset +t \supset -$ and particle b has position vector $(t) = t \subset -t^3 \supset +t$. Determine how fast b appears to be moving from a's point of view at time $t = 1$? $3\sqrt{2}$ $2\sqrt{3}$ 5 $\sqrt{4t^2+1}$ $\sqrt{5}$

Find the area of the triangle with vertices at the points $(0,0,0)$, $(1,1,2)$, and $(1,-1,4)$. $\sqrt{11}$ $\frac{11}{2}\sqrt{2}$ 8 4 $\sqrt{2}$

Find the point where the line $x = 2 - t$, $y = 1 - 2t$, $z = -2 + 3t$, intersects the plane $x - y + z = 3$. $(1,-1,1)$ $(\frac{1}{2}, \frac{3}{2}, \frac{3}{2})$ $(\frac{7}{4}, \frac{1}{2}, -\frac{5}{4})$ $(\frac{5}{4}, -\frac{3}{2}, \frac{1}{4})$ $(2,1,-2)$

Find the projection of the vector $=2 \subset -4 \supset +$ on the vector $= \subset + \supset -$. $- \subset - \supset +$ $2 \subset -4 \supset -$ $\frac{1}{7} \subset +\frac{1}{7} \supset -\frac{1}{7}$ $-2 \subset +4 \supset -$ $\frac{2}{7} \subset -\frac{4}{7} \supset +\frac{1}{7}$

Find the equation of the plane perpendicular to the line $x = 7 - 3t$, $y = 5 + 4t$, $z = -3 - t$ through the point $(1,0,1)$. $3x - 4y + z = 4$ $3x - 4y + z = 0$ $-3x + 4y - z = 6$ $-3(x-7) + 4(y-5) - (z+3) = 0$ $3x + 4y - z = 9$

Compute the volume of the box (parallelepiped) determined by the vectors $= 2 \subset - \supset +3$, $=5 \subset + \supset$, and $=2 \supset -$. 23 15 30 60 26

Find the point on the line $x = -1 + t$, $y = 1 - t$, $z = t$ closest to the origin. $(-\frac{1}{3}, \frac{1}{3}, \frac{2}{3})$ $(0, \frac{1}{2}, \frac{1}{2})$ $(0,0,1)$ $(-1,1,0)$ $(\frac{1}{3}, \frac{2}{3}, \frac{2}{3})$

Calculate the distance from the point $(2,-1,5)$ to the plane $x + 2y - z = 1$.
$\sqrt{6}$ $\frac{1}{\sqrt{6}}$ $\sqrt{27}$ 6 $\frac{1}{\sqrt{27}}$

Find a vector perpendicular to the vectors $= -3 \subset +5 \supset +9$ and $=7 \subset +$
$5 \subset +66 \supset -35$ $4 \subset +52 \supset -28$ $-3 \subset -39 \supset +21$ $-2 \subset -26 \supset +14$ $1 \subset +13 \supset -7$

Determine the equation of the line tangent to the curve $(t) = (1+t) \subset +(2+t^2) \supset +(3-t^3)$ at the point $(0,3,4)$. $x = t$, $y = 3 - 2t$, $z = 4 - 3t$ $x = 1$, $y = 2t$, $z = -3t^2$ $x = 1$, $y = 3 + 2t$, $z = 4 - 3t^2$ $x = 1 + t$, $y = 2 - 2t$, $z = 3 - 3t$ $x = 1 + t$, $y = 2 + 2t$, $z = 4 - 3t$

The total force acting on a particle of mass 2 at time t is given by $(t) = 8e^{2t} \subset +8e^{-2t} \supset$. If the particle starts at the origin with initial velocity $0 = 2 \subset -2 \supset +3$, find the position of the particle at time $t = 1$.

$(e^2 - 1) \subset +(e^{-2} - 1) \supset +3$ $(e^2 + 2) \subset +(e^{-2} - 2) \supset +3$ $(e^2 + 1) \subset +(e^{-2} - 3) \supset +3$ $e^2 \subset +e^{-2} \supset +3$ $(e^2 + 1) \subset -(e^{-2} + 1) \supset +3$

Determine which of the following curves is not smooth at some point in its domain.

$(t) = t^3 \subset +\cos(t) \supset$ $(t) = t^3 \subset +\sin(t) \supset$ $(t) = t^3 \subset +t^2 \supset +t$ $(t) = (t^3 - 3t^2) \subset +(t^2 - 2t) \supset +t^2$ $(t) = (t-1)^3 \subset +t^2 \supset$

Compute the approximate angle in radians bewteen the vectors $= 3 \subset -j + 2$ and $=2 \subset +2 \supset +$.
1.01 1.48 1.72 0.79 1.34

Determine which of the following integrals gives the length of the curve

$$(t) = t\cos(\pi t) \subset +t\sin(\pi t) \supset +, \qquad 0 \le t \le 1$$

$\int_0^1 \sqrt{1+\pi^2 t^2}\,dt$ $\int_0^1 \sqrt{1+\pi t}\,dt$ $\int_0^1 \sqrt{(1+\pi t)\cos(\pi t) + (1-\pi t)\sin(\pi t)}\,dt$ $\int_0^1 \sqrt{(1+\pi t)^2 + (1-\pi t)^2}\,dt$ $\int_0^1 \sqrt{1+(1+\pi t)^2}$ c

Determine the unit normal vector (t) of a curve given that its unit tangent is

$$(t) = \frac{t^2}{t^2+1} \subset +\frac{\sqrt{2}\,t}{t^2+1} \supset +\frac{1}{t^2+1}$$

$(t) = \frac{\sqrt{2}\,t}{t^2+1} \subset +\frac{1-t^2}{t^2+1} \supset -\frac{\sqrt{2}\,t}{t^2+1}$ $(t) = \frac{2t}{(t^2+1)^2} \subset +\frac{\sqrt{2}\,(1-t^2)}{(t^2+1)^2} \supset -\frac{2t}{(t^2+1)^2}$ $(t) = \frac{1}{2}t^2 \subset +\sqrt{2}\,t \supset +\ln(t)$ $(t) = \frac{-2t^3}{(t^2+1)^2} \subset +\frac{2\sqrt{2}\,t^2}{(t^2+1)^2} \supset -\frac{2t}{(t^2+1)^2}$ $(t) = \frac{1}{t^2+1} \subset +\frac{\sqrt{2}\,t}{t^2+1} \supset -\frac{t^2}{t^2+1}$