**Math 225: Calculus III** Name:_____

**Final Exam** *May 2, 1994* Section:_____

Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this answer sheet. There are 25 multiple choice questions worth 5 points each. You start with 25 points.

Find the area of the parallelogram with vertices $(0,0,0)$, $(1,2,-1)$, $(0,1,1)$, and $(1,3,0)$.

3.32 2.87 3.94 2.60 2.35

Find the parametric equations of the line that passes through the point $(1,0,-1)$ and is perpendicular to the plane $2x - y + 3z = 7$.

$x = 1 + 2t$
$y = -t$
$z = -1 + 3t$

$2(x-1) - y + 3(z+1) = 7$ $x = 2 + t$ $x = 2t$ $x = t$
$y = -1$ $y = -t$ $y = 0$
$z = 3 - t$ $z = 3t$ $z = -t$

Find the equation of the plane through the point $(3,1,5)$ perpendicular to the $y$-axis.

$y = 1$ $x + z = 8$ $3x + y + 5z = 9$ $x + y + z = 9$ $3(x-3) + (y-1) + 5(z-5) = 0$

A particle's position is given by $(t) = \cos(\pi t) \subset +\sin(\pi t) \supset +t^2$. Find its speed at $t = 2$.

$\sqrt{16+\pi^2}$ $-\pi \subset +\pi \supset +4$ $\sqrt{17}$ $2\pi$ $4 \supset$

Suppose a particle's acceleration is $(t) = e^t \subset +e^{-t} \supset +$. Find the particle's position at time $t = 1$ if it is initially at rest at the origin.

$(e-2) \subset +e^{-1} \supset +\frac{1}{2}$ $e \subset +e^{-1} \supset +\frac{1}{2}$ $(e-2) \subset +(e^{-1}+2) \supset +$ $e \subset +(e^{-1}+2) \supset +\frac{1}{2}$ $(e-1) \subset +(e^{-1}+1) \supset +$

Let $(t) = 2t \subset +3t^2 \supset -t^3$. Find the unit tangent vector, $(t)$, at $t = 1$. $\frac{2}{7} \subset +\frac{6}{7} \supset -\frac{3}{7}$ $2 \subset +2 \supset -3$ $2 \subset +6t \supset -3t^2$ $\frac{2}{\sqrt{4+4t^2+9t^4}} \subset +\frac{2t}{\sqrt{4+4t^2+9t^4}} \supset -\frac{3t^2}{\sqrt{4+4t^2+9t^4}}$ $\frac{2}{\sqrt{17}} \subset +\frac{2}{\sqrt{17}} \supset -\frac{3}{\sqrt{17}}$

Compute $\lim_{(x,y)\to(0,0)} \frac{x^3+y^3}{x^2+y^2}$. 0 1 2 $\frac{1}{2}$ *does not exist*

Let $f(x,y) = xe^{xy}$. Compute $f_{xy}(2,1)$. $8e^2$ $3e^2$ $2e^2$ $4e^2$ $6e^2$

The position and velocity of a smoothly moving particle at time $t = 1$ are $(1) = 3 \subset +4 \supset$ and $\check{(}1) = 5 \subset -5 \supset$, respectively. Determine how fast the distance of the particle to the origin is changing at time $t = 1$.

$-1$ $-5$ $5\sqrt{2}$ $-\sqrt{2}/2$ 35

Find the direction in which the function $f(x,y) = \sin(x^2 + xy)$ is increasing most rapidly at $(0,1)$.

$\subset \supset \subset + \supset \subset - \supset$ $2 \subset + \supset$

Find the equation of the plane tangent to the graph of $f(x,y) = x^2y - y^3$ at the point $(2,1)$.

$4x + y - z = 6$ $4x + y - z = 0$ $2xy + x^2 - 3y^2 = 5$ $4x + y = 9$ $4x + y = 0$

Find all of the critical points of the function $f(x,y) = x^2y - 6xy + 8y - x^2 + 6x - 8$.

$(2,1)$, $(4,1)$ $(2,1)$, $(3,1)$, $(4,1)$ $(2,1)$, $(2,-1)$, $(3,1)$, $(3,-1)$, $(4,1)$, $(4,-1)$ $(2,0)$, $(2,1)$, $(4,0)$, $(4,1)$, $(3,0)$, $(3,1)$ $(3,1)$

Let $f(x,y) = x^3y - 3x^2y$. determine which of the following statements is true.

$f$ has a saddle point at $(3,0)$. $f$ has a local minimum at $(3,0)$. $f$ has a local maximum at $(3,0)$. $f$ is not continuous at $(3,0)$. $(3,0)$ is not a critical point of $f$.

Determine which of the following sysytems of equations must be solved to find the extrema of the function $f(x,y) = x^2y - y^3$ subject to the constraint $x^4 + y^4 = 1$ using the method of Lagrange multipliers.

$xy = 2\lambda x^3$
$x^2 - 3y^2 = 4\lambda y^3$
$x^4 + y^4 = 1$
$2xy = \lambda x^4$
$x^2 - 3y^2 = \lambda y^4$
$x^4 + y^4 = 1$
$xy = 0$
$x^2 - 3y^2 = 0$
$x^3 + y^3 = 0$

$$\begin{aligned} xy &= 0 \\ x^2 - 3y^2 &= 0 \\ x^4 + y^4 &= 1 \\ xy &= 4\lambda x^3 \\ x^2 - 3y^2 &= 4\lambda y^3 \\ x^4 + y^4 &= 0 \end{aligned}$$

Find the area enclosed by one leaf of the rose $r = 6\sin(2\theta)$.

$9\pi/2$ $18\pi$ $9\pi$ $4\pi$ $6\pi$

Find the centroid of the triangle in the $xy$-plane with vertices $(0,0)$, $(2,1)$, $(2,0)$.

$(\frac{4}{3},\frac{1}{3})$ $(\frac{3}{4},\frac{1}{4})$ $(\frac{5}{4},\frac{1}{4})$ $(\frac{5}{4},\frac{1}{3})$ $(\frac{4}{3},\frac{1}{4})$

Compute the volume of the portion of the solid region bewteen the spheres $\rho = 1$ and $\rho = 2$ that lies inside the upper nappe of the cone $x^2 + y^2 = z^2$.

$7\pi(2-\sqrt{2})/3$ $5\pi(\sqrt{2}-1)/3$ $14\pi/3$ $5\sqrt{2}\pi/3$ $7\pi$

Find the average value of the height function $f(x,y,z) = z$ in the solid region $D$ inside the cylinder $x^2 + y^2 = 1$ between the planes $x + y + z = 4$ and $z = 0$. The volume of $D$ is $4\pi$.

2.06 2.84 2.57 3.21 2.28

Compute the Jacobian $\frac{\partial(x,y)}{\partial(u,v)}$ of the change of coordinates $u = 5x + 2y$ and $v = y - 2x$.

$\frac{1}{9}$ 9 5 $\frac{1}{5}$ $-12$

Let $\mathcal{C}$ be the curve defined by $(t) = t^3 \subset +t^2 \supset$, $0 \le t \le 1$. Compute $\int_{\mathcal{C}}(9x + 2\sqrt{y})\,ds$.

$\frac{13^{3/2}}{6}$ $\frac{17^{3/2}}{2}$ $\frac{15^{3/2}}{3}$ $\frac{11^{3/2}}{2}$ $\frac{9}{2}$

Use the Fundamental Theorem of Line Integrals to compute $\int_{\mathcal{C}}(2xy+1)\,dx+(x^2+3y^2-z)\,dy+(6z-y)\,dz$. where $\mathcal{C}$ is a smooth curve from $(0,0,0)$ to $(1,2,3)$.

32 16 8 64 4

Compute the surface area of the portion of the paraboloid $x^2 + y^2 + z = 9$ above the plane $z = 5$.

$\frac{\pi}{6}(17^{3/2} - 1)$ $\frac{\pi}{2}(11^{3/2} - 1)$ $\frac{\pi}{12}(13^{3/2} - 1)$ $\frac{\pi}{3}(15^{3/2} - 1)$ $\frac{13\pi}{3}$

Let $\Sigma$ be the part of the sphere $x^2 + y^2 + z^2 = 4$ above the triangle in the $xy$-plane with vertices $(0, 0, 0)$, $(1, 1, 0)$, and $(1, 0, 0)$. Let $= y \subset -x \supset +2x$ and let be the upward unit normal vector to $\Sigma$. Compute the flux integral $_{\Sigma} \, d\sigma$.

2/3 3/4 4/5 1/2 7/8

Let $\mathcal{C}$ be the intersection of the plane $y+z = 1$ with the cylinder $x^2+y^2 = 4$, oriented counterclockwise. Let $= x^2 \subset +xy \supset +z^2$. Use Stokes' Theorem to compute $\int_{\mathcal{C}} d$.

0 $2\pi$ $3\pi/4$ $5\pi/16$ $\pi/2$

Let $\Sigma$ be the unit sphere with outward unit normal vector . Let $= x^3 \subset +y^3 \supset +z^3$. Use the Divergence Theorem to compute the flux integral $_{\Sigma} \, d\sigma$.

$12\pi/5$ $16\pi/3$ $6\pi$ $9\pi/2$ $2\pi$