Math 225: Calculus III Name:______________________________

Exam III *April 20, 1995* Section:______________________________

Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this answer sheet. There are 15 multiple choice questions worth 6 points each. You start with 10 points.

Evaluate $\int_0^1 \int_{-x}^{2x} \int_0^{x+y+1} 2(z-1)\, dz\, dy\, dx$. $\frac{3}{4}$ $\frac{5}{6}$ $\frac{7}{8}$ $\frac{15}{16}$ $\frac{23}{24}$

Let D be the solid bounded by the planes $x+y+z=4$, $y=x$, $x=0$, and $z=0$. Find the integral that gives the volume of D. $\int_0^2 \int_x^{4-x} \int_0^{4-x-y} 1\, dz\, dy\, dx$ $\int_0^4 \int_0^y \int_0^{2-x-y} 1\, dz\, dx\, dy$ $\int_0^4 \int_0^y \int_0^{x+y+z} 1\, dz\, dx\, dy$ $\int_0^2 \int_x^2 \int_0^{2-x} 1\, dz\, dy\, dx$ $\int_0^2 \int_0^{y-x} \int_0^{2-x-y} 1\, dz\, dx\, dy$

Let D be the solid box bounded by $z=3$, $x=1$, and $y=1$ in the first octant. Suppose the density function of D is $\delta(x,y,z)=2z$. Compute the center of gravity of D. $(\frac{1}{2},\frac{1}{2},2)$ $(\frac{1}{2},\frac{1}{2},1)$ $(\frac{1}{2},\frac{1}{2},\frac{3}{2})$ $(\frac{1}{2},\frac{1}{2},\frac{1}{2})$ $(\frac{1}{2},\frac{1}{2},\frac{5}{2})$

Let D be the portion of the solid sphere of radius 3 that lies inside the cylinder $x^2+y^2=2$. Determine which of the following integrals gives $_D xyz\, dV$.

$\int_0^{2\pi} \int_0^{\sqrt{2}} \int_{-\sqrt{9-r^2}}^{\sqrt{9-r^2}} r^3 \cos(\theta)\sin(\theta) z\, dz\, dr\, d\theta$ $\int_0^{2\pi} \int_0^2 \int_{-\sqrt{3-r^2}}^{\sqrt{3-r^2}} r^2 \cos(\theta)\sin(\theta)\, dz\, dr\, d\theta$ $\int_0^{2\pi} \int_0^{\sqrt{2}} \int_{-\sqrt{9-r^2\cos^2(\theta)}}^{\sqrt{9-r^2\cos^2(\theta)}} r^2 z\, dz\, dr\, d\theta$ $\int_0^{\pi} \int_{-\sqrt{2}}^{\sqrt{2}} \int_{-\sqrt{3-r\cos(\theta)}}^{\sqrt{3-r\cos(\theta)}} r^3 \cos(\theta)\sin(\theta) z\, dz\, dr\, d\theta$ $\int_0^{2\pi} \int_{-2}^{2} \int_{-3}^{3} r^2 \cos(\theta)\sin(\theta)\, dz\, dr\, d\theta$

Convert the integral $\int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_{\sqrt{3(x^2+y^2)}}^{\sqrt{36-x^2-y^2}} (x^2+y^2+z^2)^{3/2}\, dz\, dy\, dx$ to spherical coordinates.

$\int_0^{2\pi} \int_0^{\pi/6} \int_0^6 \rho^5 \sin(\phi)\, d\rho\, d\phi\, d\theta$ $\int_0^{2\pi} \int_0^{\pi/3} \int_0^6 \rho^2 \sin(\phi)\, d\rho\, d\phi\, d\theta$ $\int_0^{\pi} \int_0^{\pi} \int_0^6 \rho^{3/2} \sin(\phi)\, d\rho\, d\phi\, d\theta$ $\int_0^{2\pi} \int_0^{\pi/2} \int_0^{36} \rho^3 \sin(\phi)\, d\rho\, d\phi\, d\theta$ $\int_0^{2\pi} \int_0^{\pi/4} \int_0^{36} \rho^4 \sin(\phi)\, d\rho\, d\phi\, d\theta$

Consider the change of variables $u=e^{x+y}$, $v=e^{x-y}$. Compute the Jacobian determinate $\frac{\partial(x,y)}{\partial(u,v)}$. $-\frac{1}{2uv}$ $e^{2x}+e^{-2y}$ 0 $\frac{1}{2}(\frac{1}{u}+\frac{1}{v})$ $e^{x+y}-e^{x-y}$

Identify which of the following plots represents the vector field $(x,y) = \frac{x}{\sqrt{x^2+y^2}} \subset -\frac{y}{\sqrt{x^2+y^2}} \supset$.

Let $(x, y, z) = x^3 y \subset +(y-z) \supset +k$. Compute . $\subset -x^3$ $3x^2y \subset + \supset - \subset +3x^2y \supset +(x^3y - 3x^2y(y-z))$ $x^3 \subset - \supset$

Let $(x, y, z) = \cos(xy) \subset + \sin(yz) \supset +(x+z)y$. Compute $\div$. $y(1-\sin(xy)) + z\cos(yz)$ $-y\sin(xy) \subset +z\cos(yz) \supset +y$ $(x+z-y\cos(yz)) \subset -y \supset +x\sin(xy)$ 0 $-\sin(xy) + \cos(xy) + y$

Let be the curve parameterized by $(t) = (t^3 - t^2) \subset +t^2 \supset, -1 \leq t \leq 1$. Determine which of the following integrals gives the line integral $\int_x +y\,ds$. $\int_{-1}^{1} t^4\sqrt{9t^2 - 12t + 8}\,dt$ $\int_{-1}^{1} t^3\,dt$ $\int_{-1}^{1} 3t^2\,dt$ $\int_{-1}^{1} t^3\sqrt{t^6 - 2t^5 + t^4}\,dt$ $\int_{-1}^{1} t^5(9t^2 - 12t + 8)\,dt$

Let be the curve parameterized by $(t) = e^{t^2} \subset +e^{-t^2} \supset +t$, $0 \leq t \leq 1$. Use the Fundamental Theorem of Line Integrals to calculate the integral $\int_y (z^2+1)\,dx + x(z^2+1)\,dy + 2z(xy+1)\,dz$. 2 1 0 $1-e^{-1}$ $e+e^{-1}+1$

Let be the counterclockwise path around the perimeter of the triangle with vertices $(0,0)$, $(1,0)$, and $(1,2)$. Use Green's Theorem to calculate $\int_(y + x\sin(y^2))\,dx + x^2 y\cos(y^2)\,dy$.

-1 0 -2 $\sin(4) - 2\cos(4)$ $\cos(4) - \sin(4) - 1$

Let be the curve parameterized by $(t) = t^2 \subset +t \supset$, $0 \leq t \leq 3$, and let $(x,y) = (x-y) \subset +(x+y) \supset$. Compute the flow integral $\int_. d$. 36 24 12 9 0

Let $f(x,y,z)$ be a function and let (x,y,z) be a vector field. Determine which of the following expressions is **not** defined. $\div$ $\div$ $\div f$ f $\div$

Which of the following gives the change from spherical coordinates (ρ, ϕ, θ) to rectangular coordinates (x,y,z).

$x = \rho\sin(\phi)\cos(\theta)$, $y = \rho\sin(\phi)\sin(\theta)$, $z = \rho\cos(\phi)$

$x = \rho\cos(\phi)\cos(\theta)$, $y = \rho\cos(\phi)\sin(\theta)$, $z = \rho\sin(\phi)$

$x = \rho\cos(\phi)$, $y = \rho\sin(\phi)$, $z = \sqrt{\rho^2 - 1}$

$x = \rho\cos(\phi)\sin(\theta)$, $y = \rho\cos(\phi)\cos(\theta)$, $z = \rho^2\sin(\phi)$

$x = \rho\cos(\phi)\cos(\theta)$, $y = \rho\sin(\phi)\sin(\theta)$, $z = \rho\tan(\phi)$