Math 225: Calculus III Name:______________________________

Final Exam *May 9, 1995* Score:______________________________

Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this answer sheet. There are 25 multiple choice questions worth 5 points each. You start with 25 points.

A vector is perpendicular to a vector $if = 0 \times = 0 + = 0 = c, somescalar c(\times) = 0$
Compute the projection of $= \subset +2$ onto $= \subset +2 \supset +$. $\frac{1}{2} \subset + \supset +\frac{1}{2} \subset +2 \supset +1 \subset +2$ $\frac{3}{5} \subset +\frac{6}{5}$ $\frac{1}{5} \subset +\frac{2}{5} \supset +\frac{1}{5}$
Determine the parametric equation of the line through the points $(-1,0,3)$ and $(2,1,-1)$. $(t) = (-1+3t) \subset +t \supset +(3-4t)$ $(t) = (-1+2t) \subset +t \supset +(3-t)$ $(t) = (2-t) \subset + \supset +(-1+3t)$ $(t) = 3t \subset -4t$ $(t) = (2+3t) \subset +(1+t) \supset +(-1+4t)$
Determine the equation of the plane perpendicular to the line $x = 3+t$, $y = 1-2t$, $z = -1+t$ at the point $(3,1,-1)$. $x-2y+z=0$ $2x-y=3$ $4x-y=11$ $3x+y-z=11$ $3x-2y+z=6$
Find $\lim_{(x,y)\to(0,0)} \frac{(x+y)^2}{x^2+y^2}$. *Does not exist* 1 $\frac{1}{2}$ 0 -1
Suppose the motion of a particle is described by $(t) = \cos(-\pi t) \subset + \sin(\pi t) \supset +t$. Find the particle's speed at time $t = 1$. 3.297 4.500 3.765 4.021 2.920
Two particles are moving along the paths

$$_1(t) = t \subset +t^2 \supset +2t$$
$$_2(t) = t^3 \subset +j + (3t-1)$$

and collide at time $t = 1$ at the point $(1,1,2)$. Find the angle in radians of this collision.

0.785 1.571 3.142 1.047 0.524
A right circular cylinder with volume V has a height, h, and a radius, r, that depend on the pressure x and temperature y of the surrounding medium. If $r = 2$, $h = 5$, $r = -2 \subset +3 \supset$, and $h = - \subset +4 \supset$, when $(x,y) = (10,20)$, which of the following gives V. $-44\pi \subset +76\pi \supset$ $20\pi \subset +4\pi \supset$ $-28\pi \subset + -4\pi$ $40\pi \subset +48\pi$
Let $f(x,y,z) = xy^2 + x^2z + z^2y$. Find the derivative of f in the direction of the vector $\subset +2 \supset +2$ at the point $(1,-1,2)$. 1 2 3 4 6
Determine the equation of the plane tangent to the graph of $f(x,y) = e^y(\sin(x) - \cos(x))$ at the point $(0,0,-1)$. $x-y-z=1$ $2e^y\sin(x) - z = 1$ $e^y(\cos(x)x + \sin(x)) - e^y(\cos(x) - \sin(x))y = 0$ $x+y+z=-1$ $x-y=0$
Let $f(x,y) = x^3 - 3xy + y^2$. Determine which of the following statements is true. f has a relative minimum at $(\frac{3}{2}, \frac{9}{4})$. f has a relative maximum at $(\frac{3}{2}, \frac{9}{4})$. f has a saddle point at $(\frac{3}{2}, \frac{9}{4})$. $(\frac{3}{2}, \frac{9}{4})$ is **not** a critical point of f *None of the above*
Find the maximum of $f(x,y) = x(1-y)$ on the unit circle $x^2+y^2=1$. 1.299 0.707 0.250 0.866 1.414
Reverse the order of integration in the iterated integral $\int_{-1}^{1}\int_{2|x|}^{2} f(x,y)$. $\int_0^2\int_{-y/2}^{y/2} f(x,y)$ $\int_{2|x|}^{2}\int_{-1}^{1} f(x,y)$ $\int_{-1}^{1}\int_0^{|y|/2} f(x,y)$ $\int_0^2\int_0^{y/2} f(x,y)$ $\int_{-2}^{2}\int_{-y/2}^{0} f(x,y)$

Determine which of the following integrals gives the volume of the solid bounded by the planes $x+y-z=0$, $x-2y+z=0$, $y=0$, and $x+y=2$.

$\int_0^{4/3}\int_{y/2}^{2-y}\int_{2y-x}^{x+y} dz\,dx\,dy$ $\int_0^{2}\int_0^{2-x}\int_{2y-x}^{x+y} dz\,dy\,dx$ $\int_0^{2}\int_0^{2-y}\int_{2y-x}^{x+y} dz\,dx\,dy$ $\int_0^{4/3}\int_{2x}^{2-x}\int_{x+y}^{2y-x} dz\,dy\,dx$ $\int_0^{2}\int_y^{2-y}\int_{x+y}^{2y-x} dz\,dx\,dy$

Let D be the solid in the first octant that is bounded above by the plane $z=9$ and is bounded below by $z=x^2+y^2$. Write ${}_D xyz\,dV$ in cylindrical coordinates. $\int_0^{\pi/2}\int_0^3\int_{r^2}^9 zr^3\cos(\theta)\sin(\theta)\,dz$ $\int_0^{\pi}\int_0^3\int_0^9 zr^2\cos(\theta)\sin(\theta)\,dz$ $\int_0^{2\pi}\int_0^1\int_0^3 zr^2\cos(\theta)\sin(\theta)\,dz$ $\int_0^{\pi/2}\int_0^3\int_r^9 zr^3\cos(\theta)\sin^2(\theta)\,dz$ $\int_0^{\pi/2}\int_0^1\int_{r^2}^3 zr^2\cos(\theta)\sin(\theta)\,dz$

Compute ${}_D\sqrt{x^2+y^2}\,dV$ where D is the solid sphere $x^2+y^2+z^2\le 4$. $4\pi^2$ π^2 $12\pi/5$ $16\pi^2/3$ $16\pi/5$

Let R be the region defined by $\frac{1}{2}x\le y\le 2x$, $1\le xy\le 2$. Using the change of variables $u=xy$, $v=\frac{y}{x}$ transform ${}_R xy\,dA$ into an iterated integral in the uv-plane. (Hint: You do not have to solve for x and y.)

$\int_{1/2}^2\int_1^2 \frac{u}{2v}\,du\,dv$ $\int_{1/2}^2\int_1^2 uv\,du\,dv$ $\int_{1/2}^2\int_1^2 2uv^2\,du\,dv$ $\int_{x/2}^{2x}\int_1^2 u\,du\,dv$ $\int_1^2\int_{x/2}^{2x}\left(uv+\frac{u}{v}\right)\,du\,dv$

Compute $\int x\,ds$ where is the curve parameterized by $(t)=\sqrt{t}\ \subset +t \supset$, $0\le t\le 1$. $(5\sqrt{5}-1)/12$ $2\sqrt{5}-1$ $(\sqrt{5}-6)/2$ $(36\sqrt{6}-1)/8$ $2\sqrt{6}-3$

Let be the curve $(t)=t\subset +t^2\supset +t$, $0\le t\le 1$. Compute $\int_x^3 z\,dx+xy\,dy+2yz^2\,dz$. 1 1/5 3/5 4/5 2/5

Let $=-z^2\subset +e^z\supset +(e^zy-2xz)$ and let $(t)=t\subset +t^2\supset +t^3$, $0\le t\le 1$. Evaluate $\int_C d$. $e-1$ 1 0 $e-e^{-1}$ $2e-1$

Use Green's Theorem to compute $\int_{\mathcal{C}}(y^3+y)\,dx+3y^2x\,dy$ where $\mathcal{C}$ is $x^2+y^2=9$ oriented counter-clockwise. -9π 3π 0 6π $-9\pi/4$

Let Σ be the portion of the plane $2x+y-z=0$ above the rectangle $0\le x\le 1$, $0\le y\le 2$ in the xy-plane. Compute ${}_\Sigma z^2-4xy\,d\sigma$.

$16\sqrt{6}/3$ $16/3$ $32\sqrt{2}$ $32\sqrt{3}$ $32/3$

Let Σ be the portion of the hyperboloid $z = x^2 - y^2$ inside the cylinder $x^2 + y^2 = 1$ with an upward pointing normal and let $(x, y, z) = 3x \subset +2y \supset +5z$. Compute $_\Sigma \cdot \, d\sigma$.

$-\pi/2$ 2π $-3\pi/2$ π $-\pi$

Let be the intersection of the paraboloid $x^2 + y^2 + z = 2$ and the plane $x + y + z = 1$ oriented counter-clockwise when viewed from above. Use Stokes' Theorem to compute $\int_x^2 \, dx + y^2 \, dy + z^2 \, dz$.

0 $\pi/3$ $2\pi/3$ $4\pi/3$ $8\pi/3$

Let $= xe^{xy} \subset -ye^{xy} \supset +z$, let Σ be the sphere $x^2 + y^2 + z^2 = 4$, and let be the outward normal vector to Σ. Evaluate the flux integral $_\Sigma \, d\sigma$. $32\pi/3$ $4\pi/3$ $8\pi/3$ $16\pi/3$ 0