Math 225: Calculus III Name:____________________________

Exam I *February 15, 1996* Section:____________________________

Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this answer sheet. There are 16 multiple choice questions worth 5 points each. You start with 20 points.

Suppose the angle between two unit vectors and $\_is30°$. Compute the dot product of and $\_$.

$\frac{\sqrt{3}}{2}$ $\frac{1}{2}$ $\frac{\sqrt{2}}{2}$ $\frac{\sqrt{3}}{2} \subset +\frac{1}{2} \supset$ $\frac{\sqrt{3}}{2} \subset -\frac{1}{2} \supset$

Find a vector perpendicular to the vectors $= 3 \subset -2 \supset +$ and $\equiv- \subset +5 \supset +$.

$-7 \subset -4 \supset +13$ $-3 \subset -10 \supset +$ $3 \subset +2 \supset +17$ $-\frac{4}{9} \subset -\frac{20}{9} \supset -\frac{4}{9}$ $-\frac{18}{7} \subset +\frac{12}{7} \supset -\frac{6}{7}$

Compute the distance from the point $(2, -1, 1)$ to the line $x = 1 - t$, $y = 3t$, $z = 4 + 2t$.

1.96 1.62 1.39 2.15 1.05

Determine the distance of the plane $4x - 5y + 2z = 6$ to the origin.

0.89 2.11 0.54 1.27 1.71

Find the volume of the box (parallelepiped) with vertices

$$(0,0,0),\ (3,1,1),\ (1,2,4),\ (-1,5,5),$$
$$(4,3,5),\ (2,6,6),\ (0,7,9),\ (3,8,10).$$

32 30 7 19 24

Which of the following sets of parametric equations describes the line in which the planes $x - 3y + 5z = 10$ and $x + y - z = 2$ intersect?

$x = 4 - t, \quad y = -2 + 3t, \quad z = 2t$ $x = -2t, \quad y = 6t, \quad z = 4t$ $x = 4 + \frac{1}{2}t, \quad y = 2 + \frac{3}{2}t, \quad z = t$ $x = 2 - 2t, \quad y = -1 - 6t, \quad z = 1 + 4t$ $x = 1 + t, \quad y = 1 - 3t, \quad z = -1 + 5t$

Determine the points in which the curve defined by

$$(t) = (t - t^2) \subset +t \supset +(t + t^2)$$

intersects the plane $x - 7y + 2z = 5$.

$(-20, 5, 30), (-2, -1, 0)$ $(20, -5, 25), (2, 1, 5)$ $(0, 0, 0), (0, 1, 3)$ $(-2, 2, -21/2), (-6, -2, -3/2)$ $(-6, 3, 16), (-12, -1, 5)$

Determine which of the following curves is not smooth at some point in its domain.

$(t) = (2t^3 - 3t^2) \subset +(t^3 - 3t) \supset +(t^2 - 2t)$ $(t) = (2t^3 + 3t^2) \subset +(t^3 + 3t) \supset +(t^2 + 2t)$ $(t) = (2t - t^2) \subset +(t^3 - 1) \supset +(t - 1)^5$ $(t) = \cos(t) \subset + \sin(t) \supset +t^2$ $(t) = (t - e^t) \subset +(t + e^{-t}) \supset +(t^3 - 3t)$

Calculate the unit normal vector (1) of a curve at $t = 1$ given that its unit tangent vector is

$$(t) = \frac{1}{1+t} \subset +\frac{\sqrt{2t}}{1+t} \supset +\frac{t}{1+t}$$

$-\frac{1}{\sqrt{2}} \subset +\frac{1}{\sqrt{2}}$ $-\frac{1}{4} \subset +\frac{1}{2\sqrt{2}} \supset +\frac{1}{4}$ $-\frac{1}{(1+t)^2} \subset +\frac{1-t}{\sqrt{2}(1+t)^2} \supset -\frac{1}{(1+t)^2}$ $-\frac{1}{4} \subset -\frac{1}{4}$ $-\frac{1}{(1+t)^2} \subset +\frac{1}{\sqrt{2t}(1+t)^2} \supset +\frac{1}{(1+t)^2}$

Suppose a particle initially at rest has acceleration given by $(t) = e^t \subset +e^{-t} \supset +2t$ at time $t \geq 0$. Compute its *speed* at time $t = 1$.

2.09 1.65 0.73 1.38 2.92

Find the equation of the plane perpendicular to the curve defined by $(t) = t^3 \subset -t^2 \supset + \sin(\pi t)$ at the point $(8, -4, 0)$. $3x - y + \frac{\pi}{4}z = 28$ $2x - y = 20$ $3x + 2y = 16$ $3(x-8) - 2(y+4) - \pi z = 0$ $3t^2x - 2ty + \pi\cos(\pi t)z = 0$

Compute the limit $\lim_{(x,y)\to(0,0)} \frac{x^2y^3 + x^3y^2}{x^5 + y^5}$. *does not exist* *exists, but has many possible values* 0 1 $\frac{2}{5}$

Suppose a particle's position at time $t \geq 0$ is described by $(t) = t\cos(t) \subset +t\sin(t) \supset +t^2$. Calculate the tangential component of the particle's acceleration.

$\frac{5t}{\sqrt{1+5t^2}}$ $\frac{4t}{\sqrt{2+4t^2}}$ $\frac{-\sin(t) \subset -\cos(t) \supset +2t}{\sqrt{1+4t^2}}$ $\frac{1}{\sqrt{5}}(-\cos(t) \subset -\sin(t) \supset +2)$ $\frac{1}{\sqrt{1+5t}}$

Which of the following represents the curve traced by $(t) = t \subset + \sin(t) \supset -t$, $0 \leq t \leq 4$?

Which of the following integrals gives the length of the curve $(t) = (t^2 - 2t) \subset +4t \supset +(t^2 + 2t)$, $0 \leq t \leq 1$?

$2\sqrt{2}\int_0^1 \sqrt{t^2+3}\,dt$ $\sqrt{2}\int_0^1 t\sqrt{t^2+12}\,dt$ $2\int_0^1 \sqrt{t+1}\,dt$ $\sqrt{2}\int_0^1 \sqrt{t(t+2)}\,dt$ $4\int_0^1 t+1\,dt$

Find the equation of the line tangent to the curve $(t) = (1-t^2) \subset +t^3 \supset +(1+t^4)$ at the point $(0,1,2)$.

$x = -2t, \quad y = 1+3t, \quad z = 2+4t$ $x = -2t, \quad y = 3t^2, \quad z = 4t^3$ $x = 1-2t, \quad y = 3t, \quad z = 1+4t$ $x = -2t, \quad y = 1+3t^2, \quad z = 2+4t^3$ $x = t, \quad y = 1, \quad z = 2+t$