Math 225: Calculus III Name:________________

Exam III *April 18, 1996* Section:________________

Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this answer sheet. There are 15 multiple choice questions worth 5 points each. You start with 25 points.

Compute the area of the region that lies inside the cardioid $r = 2 - \cos(\theta)$ above the x-axis.

$\frac{9\pi}{4}$ 2π $\frac{15\pi}{4}$ $\frac{7\pi}{2}$ $\frac{3\pi}{2}$

Let D be the solid in the first octant below the plane $2x + 2y + z = 2$. Suppose the density of D is given by $\delta(x, y, z) = 2 - z$. Which of the following integrals gives the total mass of D.

$\int_0^1 \int_0^{1-x} \int_0^{2-2x-2y} 2 - z\,dz\,dy\,dx$ $\int_0^2 \int_0^{2-z} \int_0^{2-2x-2y} 1\,dz\,dy\,dx$ $\int_0^1 \int_0^{1-x} \int_0^{2-z} 2 - 2x - 2y\,dz\,dy\,dx$ $\int_0^1 \int_0^{2-x} \int_0^{2-2y} 2 - z\,dz\,dy\,dx$ $\int_0^2 \int_0^{2-x} \int_0^{2-2x-2y} 2 - z\,dz\,dy\,dx$

Find the volume of the solid bounded above by the paraboloid $x^2 + y^2 + z = 4$ and below by the xy-plane.

8π $\frac{32\pi}{3}$ 16π $\frac{9\pi}{2}$ $\frac{25\pi}{3}$

Let D be the solid bounded above by $\rho = 2\sin(\phi)$ and below by $\phi = \pi/2$ in spherical coordinates. Compute ${}_D z\,dV$.

$4\pi/3$ $2\pi/3$ π 2π $\pi^2/2$

Let R be the region bounded by the lines

$$\begin{aligned} x + y &= 1, & x + y &= 2 \\ x - 2y &= -2, & x - 2y &= 1 \end{aligned}$$

Determine which of the following integrals gives the value of $_R xy\, dA$ after the substitution $u = x + y$, $v = x - 2y$.

$\int_{-2}^{1}\int_{1}^{2} \frac{1}{27}(u-v)(2u+v)\, du\, dv$ $\int_{-2}^{1}\int_{1}^{2} \frac{1}{9}(v-u)(2u+v)\, du\, dv$ $\int_{-2}^{1}\int_{1}^{2}(x+y)(x-2y)\, dx\, dy$ $\int_{1}^{2}\int_{-2}^{1} \frac{1}{3}(x+y)(2y-x)\, dy\, dx$ $\int_{1}^{2}\int_{-2}^{1} \frac{1}{3}uv\, du\, dv$

Determine which of the following represents the vector field $(x, y) = -\frac{y}{5} \subset +\frac{x}{5} \supset$.

Let $(x, y) = (x^3 + yz) \subset +(y^3 + xz) \supset +(z^3 + xy)$. Compute $\div$.

$3(x^2 + y^2 + z^2)$ $(3x^2 + y + z) \subset +(3y^2 + x + z) \supset +(3z^2 + x + y)$ $3x^2 \subset +3y^2 \supset +3z^2$ $3(x^2 + y^2 + z^2) + 2(x + y + z)$ 0

Let $(x, y) = e^{-y}\cos(x) \subset +e^{-y}\sin(x) \supset +z$. Compute .

$2e^{-y}\cos(x)$ $-2e^{-y}\sin(x)$ $1 - 2e^{-y}\sin(x)$ $-e^{-y}\cos(x) \subset -e^{-y}\cos(x) \supset$

Calculate the line integral $\int_C x\, ds$ where C is the curve parameterized by $(t) = t \subset +t^2 \supset +t$, $0 \le t \le 1$.

0.989 2.637 1.905 3.872 3.223

Let C be the space curve parameterized by $(t) = (t^2 - 1) \subset +t^3 \supset +(t-1)$, $0 \le t \le 1$. Evaluate $\int_C z\, dx + x\, dy + y\, dz$.

-0.483 -0.733 -0.5 -1.235 -0.822

Let C be the plane curve parameterized by $(t) = \sqrt{1 + \cos(t)} \subset +\sqrt{1 + \sin(t)} \supset$, $0 \le t \le \pi$, and let $f(x, y) = (x^2 + y^2)^{x^2 y^2}$. Calculate $\int_C (f) \cdot d$.

-8 $\sqrt{2} - 1$ 0 $-\sqrt{2}$ 4

Let C be the boundary of the upper half of a circle of radius 1. Use Green's Theorem to evaluate $\int_C xy\, dx + (x^2 + y^2)\, dy$

0 $-1/3$ $-1/2$ $-3/4$ $-2/3$

Evaluate the iterated integral $\int_0^{\sqrt{\pi}} \int_0^z \int_0^{yz} \cos(y^2)\, dx\, dy\, dz$.

$\frac{1}{2}$ $\frac{\pi}{2}$ $\sqrt{\pi}$ $\frac{1}{4}$ $\frac{\pi-1}{2}$

Compute ${}_R e^{-x^2-y^2}\, dA$, where R is the region defined by $x^2+y^2 \leq 2$.

$\pi(1-e^{-2})$ $2\pi e^{-2}$ 4π $2\pi(e^{-2}-1)^2$ $(1-e^{-2})/2$

Find the average value of the function $f(x,y) = xy^2 - zx^2$ in the solid region defined by $0 \leq x \leq 2$, $0 \leq y \leq 3$, and $0 \leq z \leq 1$.

$7/3$ $1/3$ $5/6$ $3/2$ $11/6$