Math 225: Calculus III Name:______________________________

Final Exam *May 8, 1996* Section:______________________________

Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this answer sheet. There are 25 multiple choice questions worth 5 points each. You start with 25 points.

Let $= -2 \subset +4 \supset +5$ and $= \subset +3 \supset -2$. Compute $\cdot$.

0 5 -2 $-23 \subset + \supset +10$ $-2 \subset +12 \supset -10$

Let $=\subset -3$ and $=2 \subset - \supset +$. Compute $\times$.-3ı- 7ȷ- -3ı- ȷ+ 555-1

Determine the parametric equations of the line through the point $(1, -2, 1)$ parallel to the vector $= 4 \subset + \supset -3$.

$x = 1 + 4t$

$y = -2 + t$

$z = 1 - 3t$

$x = 4 + t$

$y = 1 - 2t$

$z = -3 + t$

$x = 4t$

$y = t$

$z = -3t$

$x = 1 + t$

$y = -2 + t$

$z = t$

$x = 1 - 4t$

$y = 2 - t$

$z = 3t$

Determine the equation of the plane perpendicular to the vector $=\subset +3 \supset -5$ passing through the point $(4, -1, 2)$.

$x + 3y - 5z = -9$ $4x - y + 2z = 0$ $4x - y + 2z = -9$ $(x + 4) + 3(y + 1) - 5(z + 2) = 0$ $x + 3y - 5z = 0$

Determine which of the following integrals gives the length of the curve parameterized by
$(t) = e^{-t}\cos(t) \subset +e^{-t}\sin(t) \supset -te^{-t}$, $0 \le t \le 1$.

$\int_0^1 e^{-t}\sqrt{2 + (1-t)^2}\,dt$ $\int_0^1 \sqrt{e^{-2t} + 1}\,dt$ $\int_0^1 \sqrt{e^{-t} + (1-t)^2}\,dt$ $\int_0^1 e^{-t}\sqrt{(1+\cos(t))^2 + (1+\sin(t))^2 +$
$\int_0^1 e^{-t}\sqrt{2\cos(t)\sin(t) + (1-t)^2}\,dt$

Find the parametric equations of the line tangent to the curve $(t) = \log(1/t) \subset +t \supset +t^2$ at the point $(0, 1, 1)$.

$x = -t$

$y = 1 + t$

$z = 1 + 2t$

$x = 1 - 1/t$

$y = 3$

$z = 1 + 2t$

$x = -1$

$y = 1 + t$

$z = 1 + 2t^2$

$x = -1/t$

$y = 1 + t$

$z = 1 + 2t^2$

$x = -t$

$y = t$

$z = 2t$

Let $f(x, y) = x^3 e^{y^2} + \sin(xy)$. Compute f_{xy}. $6x^2 y e^{y^2} + \cos(xy) - xy\sin(xy)$ $(3x^2 + 4x^3y^2)e^{y^2} - x^2\sin(xy)$ $(3x^2 + 2x^3y)e^{y^2} + (x+y)\cos(xy)$ $(3x^2e^{y^2} + y\cos(xy)) \subset +(2x^3ye^{y^2} + x\cos(xy)) \supset 6x^2ye^{y^2} - xy\cos(xy)$

Let $f(x, y) = x\sqrt{x^2 + y^2}$ and suppose x and y are functions of u and v. If $x/du = 3$, $x/dv = -2$, $y/du = -4$ and $y/dv = 7$ when $(x, y) = (1, 2)$, compute f/du at that point. $2\sqrt{5}$ $4/\sqrt{5}$ $-1/\sqrt{5}$ $4\sqrt{5}$ $2/\sqrt{5}$

Calculate the derivative of the function $f(x, y) = x^3y - y^2$ in the direction $30°$ above horizontal at the point $(1, -1)$. -1.098 0 -2.130 -1.703 -2.517

Determine the equation of the plane that is tangent to the graph of $f(x, y) = x^3 + y^3$ at the point $(-1, 2, 9)$. $3x + 12y - z = 12$ $3x + 12y = 21$ $3x + 12y - z = 0$ $3x^3 + 3y^3 + z = 0$ $3x^2(x + 1) + 3y^2(y - 2) + (z - 9) = 0$

Determine which of the following statements describes the graph of $f(x, y) = 4x^3 - 6x^2y + 3y^2$ over the point $(1, 1)$.

a saddle point a local maximum a local minimum not a critical point not continuous

Find the maximum of $f(x, y) = x^4y$ subject to the constraint $x^2 + 5y^2 = 5$.

7.155 10.062 11.180 6.708 8.944

Reverse the order of integration in the double integral $\int_{-3}^{3}\int_{1}^{10-x^2} f(x, y)\,dy\,dx$. $\int_{1}^{10}\int_{-\sqrt{10-y}}^{\sqrt{10-y}} f(x, y)\,d$ $\int_{1}^{10-x^2}\int_{-3}^{3} f(x, y)\,dx\,dy$. $\int_{-3}^{3}\int_{1}^{10-y^2} f(x, y)\,dx\,dy$. $\int_{1}^{10}\int_{0}^{\sqrt{10-y}} f(x, y)\,dx\,dy$ $\int_{0}^{10}\int_{-\sqrt{10-y}}^{\sqrt{10}} f(x, y)\,dx\,dy$

Calculate the area of the region bounded above by the circle of radius 1 and below by the portion of the spiral $r = \theta/\pi$ above the x-axis in polar coordinates.

$\frac{1}{3}\pi$ $\frac{1}{2}\pi - \frac{1}{6}\pi^3$ $\frac{1}{6}\pi^2$ $\frac{1}{2}\pi$ $\frac{1}{3}\pi^2 - \frac{1}{6}\pi^3$

Which of the following integrals gives the volume of the solid bounded by the xy-plane, the yz-plane, the parabolic sheet $x + y^2 = 1$, and the cylinder $y^2 + z^2 = 1$.

$\int_0^1 \int_{-\sqrt{1-x}}^{\sqrt{1-x}} \int_0^{\sqrt{1-y^2}} 1\, dz\, dy\, dx$ $\int_0^1 \int_0^{1-y^2} \int_0^{\sqrt{1-y^2}} 1\, dz\, dx\, dy$ $\int_{-1}^1 \int_{1-y^2}^1 \int_0^{\sqrt{1-y^2}} 1\, dz\, dx\, dx$ $\int_0^1 \int_{y^2-1}^{1-y^2} \int_0^{x+y^2} 1\, dz\, dx\, dy$ $\int_{-1}^1 \int_0^{x+y^2} \int_0^{y^2+z^2} 1\, dz\, dy\, dx$

Let D be the part of the cylinder $x^2 + y^2 = 4$ below the plane $y + z = 2$ and above the xy-plane. The volume of D is 8π. Compute the z-coordinate of the centroid of D.

1.25 1.33 1.50 0.67 1.75

Determine which of the following integrals gives the volume of the part of the solid sphere $x^2 + y^2 + z^2 \leq 1$ cut out by the the upper nappe of the cone $3z^2 = x^2 + y^2$.

$\int_0^{2\pi} \int_0^{\pi/3} \int_0^1 \rho^2 \sin(\phi)\, d\rho\, d\phi\, d\theta$ $\int_0^{2\pi} \int_0^{\pi/6} \int_0^1 \rho^2 \sin(\phi)\, d\rho\, d\phi\, d\theta$ $\int_0^{2\pi} \int_0^{\sin(\phi)} \int_0^{\rho^2} 1\, d\rho\, d\phi\, d\theta$ $\int_0^{2\pi} \int_0^{\pi/4} \int_0^1 \rho$ si $\int_0^{2\pi} \int_0^{\pi/2} \int_0^1 \rho^2 \cos(\phi)\, d\rho\, d\phi\, d\theta$

Consider the change of variables $u = \log(x^2 + y^2)$, $v = y/x$. Compute the Jacobian determinant $\frac{\partial(x,y)}{\partial(u,v)}$.

$\frac{e^u}{2(1+v^2)}$ $2e^{-u}(1-v^2)$ $e^{-u}(1+v^2)$ $\frac{e^u}{(1-v^2)}$ $\frac{2(x^2-y^2)}{x^2+y^2}$

Compute the line integral $\int_z dx + (x-1)z\,dy - z(x+y))\,dz$ where is the curve parameterized by $(t) = e^t \subset -e^{-t} \supset +t$, $0 \le t \le 2$.

2 0 $2e^2 - e^{-2}$ $(e^2 - e^{-2})/2$ $(3e^2 - e^{-2})/2$

Use the Fundamental Theorem of Line Integrals to calculate

$$\int_(x^2 + y^2 + z^2)\,dx + 2y(x - z^3)\,dy + z(2x + 3z - 3y^2 z)\,dz$$

where is the curve parameterized by

$$(t) = \cos[2\pi(1+t^2)] \subset +(1-t^2) \supset +t, \qquad 0 \le t \le 1$$

1 -1 0 $4/3$ $-7/3$

Let be the boundary of the triangle with vertices

$$(0,0), \qquad (1,0), \qquad (0,2)$$

oriented counter-clockwise. Use Green's Theorem to compute the line integral

$$\int_(x^3 + xy^2)\,dx + (y^3 + x^2 y)\,dy$$

0 $2/3$ -1 $10/3$ 1

Calculate the surface area of the part of the paraboloid $x^2 + y^2 + z = 9$ above the xy-plane.

$\frac{\pi}{6}[37^{3/2} - 1]$ $9\sqrt{9-\pi^2}$ $\frac{\pi}{3}[(10)^{3/2} - 1]$ $\frac{\pi\sqrt{3}}{2}$ $3\sqrt{1+3\pi^2}$

Let Σ be the portion of the sphere of radius 2 in the first octant and let be the unit upward normal vector to Σ. Compute the flux integral $_\Sigma(xz \subset +y^2) \cdot \, d\sigma$.

2π π 0 $\frac{3\pi}{2}$ $\frac{\pi}{2}$

Let be the boundary of the triangle cut out from the plane $x + y + z = 1$ by the first octant. Use Stokes' Theorem to calculate the line integral $\int_(z \subset +x \supset +y) \cdot d$

$\frac{3}{2}$ 3 $\frac{1}{2}$ 0 2

Let D be the solid rectangular box defined by $0 \le x \le 3$, $0 \le y \le 2$, and $0 \le z \le 1$. Let Σ be the boundary of this box and let be its outward unit normal vector. Use the Divergence Theorem to express the flux integral

$$_{\Sigma}[(x^2y + z) \subset +(y^2z - x) \supset +(xz^2 + y)] \cdot d\sigma$$

as a triple integral.

$2\int_0^1 \int_0^2 \int_0^3 (xy + yz + xz)\, dx\, dy\, dz$ $\int_0^1 \int_0^2 \int_0^3 0\, dx\, dy\, dz$ $\int_0^1 \int_0^2 \int_0^3 (x^2y + y^2z + xz^2 - x + y + z)\, dx\, dy\, dz$ $\int_0^1 \int_0^2 \int_0^3 \frac{x\subset + y\supset + z}{xz}\, dx\, dy\, dz$ $2\int_0^1 \int_0^2 \int_0^3 (x \subset + y \supset + z)\, dx\, dy\, dz$