Math 225: Calculus III Name:____________________________

Exam II *March 20, 1997* Section:____________________________

Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this answer sheet. There are 15 multiple choice questions worth 5 points each. You start with 25 points.

Determine which of the following statements is true about the function
$f(x,y) = ye^x + xe^y$

$f(x,y)$ does *not* have a critical point at $(-1,-1)$.

$f(x,y)$ has a local minimum at $(-1,-1)$.

$f(x,y)$ has a local maximum at $(-1,-1)$.

$f(x,y)$ has a saddle point at $(-1,-1)$.

none of the above

Let $f(x,y) = (1+x^2)^{\sin(x)} + xy^2$. Compute f_{xy}.

$2y$

$2xy$

$(1+x^2)^{\sin(x)} \log(1+x^2)2x + 2x$

$\sin(x)(1+x^2)^{\sin(x)-1} + 2y$

$(1+x^2)^{\sin(x)-1}(\sin(x) + 2x\cos(x)\log(1+x^2)) + 2xy$

Suppose $z = x^2\cos(1+y)$ and $x = u^3\log(v)$, $y = v^2 - u^2$. Determine which of the following expressions gives ẓ$/du$.

$6xu^2\cos(1+y)\log(v) + 2x^2u\sin(1+y)$

$\frac{2xu}{v}\cos(1+y) - 2x^2v\sin(1+y)$

$2x\cos(1+y) - x^2\sin(1+y)$

$2xu^3\cos(1+y)\log(v) - x^2\sin(1+y)(v^2-u^2)$

$6xu^2\cos(1+y)\log(v) - 2x^2v\sin(1+y)$

Find the direction in which the function $f(x,y,z) = xy^3 - x^2z^2$ increases most rapidly at the point $(2,-1,3)$.

$-37 \subset +6 \supset -24$

$- \subset +3 \supset -6$

$-4 \subset +11 \supset -5$

$-19 \subset +12 \supset -4$

$-29 \subset +15 \supset -2$

Let $f(x,y) = e^{xy} - x^2 - y^2$. Considering all possible unit direction vectors $\breve{}$, find the largest value attained by D

f$atthepoint$(0,2).

$2\sqrt{5}$

2

$2\sqrt{3}$

4

$2\sqrt{2}$

Find the equation of the line perpendicular to the curve $x^3 - 3y^3 = 3$ at the point $(3,2)$.

$4x + 3y = 18$

$4x - 3y = 6$

$3x - 4y = 1$

$3x + 4y = 17$

$9x + 8y = 43$

Find the equation of the plane tangent to the surface defined by $x^2 - xy + yz = 5$ at the point $(1, 2, 3)$.

$y + z = 5$

$(2x - y)(x - 1) + (z - x)(y - z) + y(z - 3) = 0$

$x + y + z = 6$

$(x - 1) + 2(y - 2) + 6(z - 3) = 0$

$x + 2y + z = 10$

Find the critical points of the function $f(x, y) = 8(x^2 + y^2) - \frac{1}{xy}$.

$(1/2, -1/2), (-1/2, 1/2)$

$(1/2, 1/2)$

$(1/2, 1/2), (-1/2, 1/2), (1/2, -1/2)$

$(1/2, 1/2), (-1/2, 1/2), (1/2, -1/2), (-1/2, -1/2)$

$(1/2, 1/2), (-1/2, -1/2)$

Determine which of the graphs below corresponds to the function

$$f(x, y) = 2x^3y - y^2 - 6x$$

As a particle moves around an elliptical path, $x^2 + 2y^2 = 1$, its height above the ground is given by $2 + 6x^2y + 2y^3$. Compute the maximum height the particle achieves.

3.789

2.707

0.211

1.293

4.211

Find the maximum value of the function $f(x, y) = (x^2 - x)(y^2 - y)$ on the rectangle $0 \leq x \leq 1$, $0 \leq y \leq 1$.

0.0625

0.125

0.5

0.25

0.0

The pressure at a point (x, y) on an square plate, $0 \leq x \leq a$, $0 \leq y \leq a$, is given by $P(x, y) = xy(a - x)$. Find the average pressure on the plate.

$a^3/12$

$a^5/4$

$a^5/6$

$a^3/2$

$a/3$

Evaluate the double integral $\int_0^{\sqrt{\pi}} \int_0^{\sqrt{x}} \sin(x^2) y \, dy \, dx$.

1/2

1

1/3

1/4

0

Determine which of the following double integrals gives the volume of the region in the first octant that lies under the plane $3x + 2y + z = 4$.

$\int_0^{4/3} \int_0^{2-3x/2} 4 - 3x - 2y \, dy \, dx$

$\int_0^4 \int_0^{3x+2y} z \, dx \, dy$

$\int_0^2 \int_0^{4-2x} 4 - 3x - 2y \, dx \, dy$

$\int_0^2 \int_0^{4-2y} (4 - 2y - z)/3 \, dx \, dy$

$\int_0^{4/3} \int_0^2 3x + 2y \, dy \, dx$

Reverse the order of integration in the double integral $\int_0^4 \int_{-\sqrt{4-y}}^{\sqrt{4-y}} f(x, y) \, dx \, dy$.

$\int_{-2}^2 \int_0^{4-x^2} f(x, y) \, dy \, dx$

$\int_{-\sqrt{4-y}}^{\sqrt{4-y}} \int_0^4 f(x, y) \, dy \, dx.$

$\int_0^2 \int_{-(4+x)^2}^{(4+x)^2} f(x, y) \, dy \, dx$

$\int_{-4}^4 \int_0^{4-x^2} f(x, y) \, dy \, dx$

$\int_0^2 \int_0^{4+x^2} f(x, y) \, dy \, dx$