Math 225: Calculus III Name:________________________

Exam III *April 17, 1997* Section:________________________

Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this answer sheet. There are 14 multiple choice questions worth 5 points each. You start with 30 points.

Calculate the area bounded by the spirals $r = \theta$ and $r = 2\theta$ and the positive x-axis.

$4\pi^3$ $\pi^2/2$ $2\pi^2$ $\pi^3/2$ π^2

Let D be the solid region bounded by the planes $x + 2y + z = 3$, $2x + y - z = 3$, $x = 0$, and $y = 0$. Determine which of the following integrals gives ${}_D f(x, y, z)\, dV$.

$\int_0^2 \int_0^{2-x} \int_{2x+y-3}^{3-x-2y} f(x,y,z) dzdydx$ $\int_0^x \int_0^y \int_{2x+y+z}^{x+2y+z} f(x,y,z) dzdydx$ $\int_0^3 \int_0^{3-2x} \int_0^{3-x-2y} f(x,y,z) dzdydx$ $\int_0^{3/2} \int_0^{(3-x)/2} \int_0^{3-x-2y} f(x,y,z) dzdydx$ $\int_0^3 \int_{3-2x}^{(3-x)/2} \int_{2x+y-3}^{3-x-2y} f(x,y,z) dzdydx$

Evaluate the iterated integral $\int_0^1 \int_0^z \int_0^y e^{x-y} dx\, dy\, dz$.

$\frac{1}{2} - e^{-1}$ $1 - e^{-1}$ $e - \frac{1}{2}$ $\frac{3}{2} + e^{-1}$ $e^{-1} - 1$

Consider the region defined by the inequalities $2x-1 \leq y \leq 2x$ and $-x^2 \leq y \leq 4-x^2$

Using the change of variables

$$\begin{array}{ll} u = 2x - y & x = -1 + \sqrt{1+u+v} \\ v = x^2 + y & y = \sqrt{1+u+v} - u - 2 \end{array}$$

transform the integral $_R(x+1)^2\,dA$ into an iterated integral in u and v.

$\frac{1}{2}\int_0^4\int_0^1 \sqrt{1+u+v}\,du\,dv$ $\int_0^4\int_0^1 \sqrt{1+u+v}\,du\,dv$ $\frac{1}{2}\int_0^1\int_0^4 1{+}u{+}v\,du\,dv$ $\int_0^1\int_0^4 \sqrt{1+u+v}\,du\,dv$ $\int_0^4\int_0^1 1+u+v\,du\,dv$

Suppose the density of the solid unit sphere $x^2{+}y^2{+}z^2 \leq 1$ is given by $\delta(x,y,z) = 1{+}z$. Find its center of mass.

$(0,0,\frac{1}{5})$ $(0,0,0)$ $(0,0,\frac{1}{10\pi})$ $(0,0,\frac{2}{15})$ $(0,0,\frac{3}{4\pi})$

Find the vector field that is represented in the following plot.

$\frac{x}{\sqrt{x^2+y^2}} \subset -\frac{y}{\sqrt{x^2+y^2}} \supset$ $\frac{x}{\sqrt{x^2+y^2}} \subset +\frac{y}{\sqrt{x^2+y^2}} \supset$ $\frac{y}{\sqrt{x^2+y^2}} \subset +\frac{x}{\sqrt{x^2+y^2}} \supset$ $\frac{y}{\sqrt{x^2+y^2}} \subset -\frac{x}{\sqrt{x^2+y^2}} \supset$ $\frac{x+y}{\sqrt{2(x^2+y^2)}} \subset -\frac{x-y}{\sqrt{2(x^2+y^2)}} \supset$

Determine which of the following sets of inequalities describes the solid region bounded above by the sphere $x^2+y^2+z^2 = 8$ and below by the paraboloid $2z = x^2+y^2$.

$\frac{1}{2}r^2 \leq z \leq \sqrt{8-r^2}$
$0 \leq r \leq 2$
$0 \leq \theta \leq 2\pi$

$0 \leq \rho \leq 2\sqrt{2}$
$0 \leq \phi \leq \pi/4$
$0 \leq \theta \leq 2\pi$

$0 \leq z \leq \sqrt{8 - r^2}$
$0 \leq r \leq 4$
$0 \leq \theta \leq \pi$

$\frac{1}{2}r \leq \rho \leq 2\sqrt{2}$
$0 \leq \phi \leq \pi/2$
$0 \leq \theta \leq \pi$

$\frac{1}{2}r^2 \leq z \leq 2\sqrt{2}$
$0 \leq r \leq \sqrt{2}$
$0 \leq \theta \leq 2\pi$

Calculate the volume of the solid region in the first octant that lies inside the cylinder, $x^2 + y^2 = 1$, and below the plane, $x + y + z = 2$.

$\frac{\pi}{2} - \frac{2}{3}$ $\pi - \frac{1}{3}$ $2\pi - \frac{4}{3}$ $\frac{\pi-2}{3}$ $\frac{\pi-1}{2}$

Let $(x, y, z) = xy^3 \subset +(x^2 + z^2) \supset +x^3 z$. Compute $\div$.

$x^3 + y^3$ $-2z \subset -3x^2 z \supset +(2x - 3xy^2)$ $(3x^2 y + y^3) \subset +2(x + z) \supset +(x^3 + 3x^2 z)$ $x^3 + y^3 + 3xy^2 + 3x^2 z + 2x + 2z$ $-2 \subset -3x(x + 2z) \supset +(2 - 6xy - 3y^2)$

Let $= (x^2 + y^3 z) \subset +(xz - y) \supset +x^2 yz$. Compute .

$x(xz-1) \subset +y(y^2 - 2xz) \supset +z(1-3y^2)$ $x^2 y + 2x - 1$ $(y^3 + 3y^2 z + 2x) \subset +(x + z - 1) \supset +x(xy + xz + 2yz)$ $2x \subset - \supset +x^2 y)$ $\mathbf{0}$

Let be the directed line segment from the point $(-1, 0, 4)$ to the point $(3, 2, 0)$. Compute the value of line integral $\int_(x - 2y)z\, ds$.

-12 -6 $1/2$ $1/3$ 1

Compute the flow integral $\int_. d$ of the velocity field $(x, y, z) = z \subset +x \supset +y$ along the curve $(t) = t \subset +t^2 \supset +t^3$, $-1 \leq t \leq 1$.

2.53 1.42 3.11 0.87 2.00

Use the Fundamental Theorem of Line Integrals to evaluate

$$\int_{(} 2xy + z)\, dx + (x^2 + 3y^2 + z)\, dy + (x + y - 1)\, dz$$

where is the curve $(t) = 3te^{(t-1)^3} \subset +2te^{(t-1)^2} \supset +te^{(t-1)}$, $0 \leq t \leq 1$.

30 12 10 6 5

Let R be the rectangular region defined by $-1 \leq x \leq 2$, $-2 \leq y \leq 3$ and let be the boundary of R oriented counterclockwise. Use Green's Theorem to compute $\int_x y\, dx - (x^2 + y^2)\, dy$

-22.5 -18.0 15.5 29.0 -14.5