Math 225: Calculus III Name:__________________________

Final Exam *May 8, 1997* Section:__________________________

Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this answer sheet. There are 25 multiple choice questions worth 5 points each. You start with 25 points.

Compute $\lim_{(x,y)\to(0,0)} \frac{y+xy}{x^2+xy^2+y}$ 1 0 1/2 2 *does not exist*

Let $(x, y, z) = \frac{x}{y} \subset + \frac{y}{z} \supset + \frac{z}{x}$. Compute . $\frac{y}{z^2} \subset + \frac{z}{x^2} \supset + \frac{x}{y^2}$ $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$ $\frac{-y\subset+z\supset-x}{x^2+y^2+z^2}$ $\frac{x^3z^2+x^2y^3+y^2z^3}{x^2y^2z^2}$ $\frac{1}{x} \subset + \frac{1}{y} \supset + \frac{1}{z}$

Compute the angle between the planes $2x + y + z = 1$ and $x + 2y - z = 1$. $\pi/3$ 0 $\pi/2$ $\pi/4$ $3\pi/4$

Determine the equation of the plane through the point $(3, 0, 1)$ that is parallel to the vectors $\subset + \supset -$ and $\subset - \supset$.

$x + y + 2z = 5$ $x + y - z = 2$ $x - y = 3$ $x + 2y + z = 4$ $x - y - 2z = 1$

Suppose a particle's position is given by $r(t) = e^{t^2-1} \subset +e^{1-t^2} \supset +t$. Determine the particle's speed at the point $(1, 1, -1)$. 3 2 $\subset -2 \supset +$ $\sqrt{4e^2 + 4e^{-2} + 1}$ 1

Consider the curve parameterized by $r(t) = \frac{t^2}{2} \subset +t \supset -t$. Calculate the unit normal vector to this curve at the point $(1, \sqrt{2}, -\sqrt{2})$. $\frac{1}{\sqrt{2}} \subset -\frac{1}{2} \supset +\frac{1}{2}$ $\frac{1}{\sqrt{2}} \subset +\frac{1}{2} \supset -\frac{1}{2}$ $\frac{1}{\sqrt{2}} \subset +\frac{1}{2} \supset +\frac{1}{2}$ $\frac{1}{\sqrt{2}} \subset -\frac{1}{2} \supset -\frac{1}{2}$ $\frac{-1}{\sqrt{2}} \subset -\frac{1}{2} \supset +\frac{1}{2}$

Let $f(x, y, z) = \frac{x^2-yz}{xy^2}$. Compute f_{xyz} $\frac{-1}{x^2y^2}$ $\frac{-2}{x^2y^3}$ $\frac{-2}{x^3y^2}$ $\frac{-4}{x^3y^3}$ $\frac{-1}{xy}$

If $z = \cos(x^2 - y^2)$ and $x = e^u$, $y = ue^{-v}$, determine which of the following expressions gives z/du.

$-2(xe^u - ye^{-v})\sin(x^2 - y^2)$ $2(e^{2u} - e^{-2v})\sin(e^{2u} - e^{-2v}u^2)$ $-2u^2e^{-2v}\sin(x^2 - y^2)$ $-2e^{2u}\sin(x^2 - y^2)$ $-2(x - y)\sin(x^2 - y^2)$

Calculate the derivative of the function $f(x, y, z) = xy^2 - z^3$ at $(-1, 2, 1)$ in the direction of the vector $(2, 1, 2)$. $-2/3$ $1/3$ -2 1 0

Determine the equation of the plane tangent to the surface $z^2 = x^3 + y^3$ at the point $(1, 2, 3)$.

$x + 4y - 2z = 3$ $(x - 1) + 2(y - 4) + 3(z + 2) = 0$ $(x - 1) + 4(y - 2) = 0$ $x + 2y = 0$ $3x + 4y - 6z = 9$

Determine which of the following statements describes the function

$$f(x, y) = y^2(x - 1) + x^2$$

$f(x, y)$ has a saddle point at $(0, 0)$. $f(x, y)$ has a local minimum at $(0, 0)$. $f(x, y)$ has a local maximum at $(0, 0)$. $f(x, y)$ has a critical point at $(1, \sqrt{2})$. $f(x, y)$ does not have a critical point at $(0, 0)$.

Find the maximum of the function $f(x, y) = x(y + 1)$ subject to the constraint $2x^2 + y^2 = 1$.

0.919 1.431 1.564 1.111 1.333

Find all the critical points of the function $f(x, y) = x^3y - xy + 3x^2 + 2x$. $(0, 2)$, $(1, -4)$, $(-1, 2)$ $(0, 2)$, $(1, -4)$, $(1, 4)$, $(-1, 2)$ $(0, 2)$, $(1, -4)$, $(-1, 2)$, $(1, 2)$ $(0, 2)$, $(-1, 4)$, $(1, -2)$ $(0, 2)$, $(-1, -4)$, $(1, -4)$, $(-1, 2)$, $(1, 2)$

Reverse the order of integration in the integral $\int_1^2 \int_{\sqrt{x-1}}^{\sqrt[3]{x-1}} f(x,y)\,dy\,dx$.

$\int_0^1 \int_{y^3+1}^{y^2+1} f(x,y)\,dx\,dy$ $\int_{\sqrt{x-1}}^{\sqrt[3]{x-1}} \int_1^2 f(x,y)\,dx\,dy$. $\int_1^2 \int_{y^2+1}^{y^3+1} f(x,y)\,dx\,dy$ $\int_0^1 \int_{1+\sqrt[3]{y}}^{1+\sqrt{y}} f(x,y)\,dx\,dy$
$\int_0^2 \int_{y^2}^{y^3} f(x,y)\,dx\,dy$

Find the area of the region below the curve $r = \cos(\theta) - \sin(\theta)$ and above the x-axis.

$\frac{\pi-2}{8}$ $\frac{\pi-1}{4}$ $\frac{3\pi-2}{4}$ $\frac{\pi-3}{4}$ $\frac{2\pi-3}{8}$

Find the average height above the xy-plane of the points in the solid region of the first octant below the inverted cone $z = 2 - \sqrt{x^2+y^2}$. The volume of the solid is $2\pi/3$.

1/2 2/3 $\pi/2$ $\pi/3$ 7/8

Let D be the solid region between the spheres of radii 1 and 2. Compute the integral $_D z^2\,dV$.

$\frac{124\pi}{15}$ $\frac{12\pi}{5}$ 24π $\frac{8\pi}{3}$ 4π

Consider the change of variables $u = \sqrt{x+y}$, $v = \sqrt{x-y}$. Compute the Jacobian determinant $\frac{\partial(x,y)}{\partial(u,v)}$.

$-2uv$ $\frac{1}{2uv}$ $\frac{1}{4\sqrt{x^2-y^2}}$ $-\frac{1}{2\sqrt{x+y}}$ $\frac{1}{4}$

Let be the semi-circle $y = \sqrt{1-x^2}$ and let be the unit tangent vector to oriented counterclockwise. Determine which of the following integrals gives the value of $\int \,ds$ where $(x,y) = y^2 \subset +x^2 \supset$.

$\int_0^\pi \cos^3(t) - \sin^3(t)\,dt$ $-\int_0^\pi \cos(t)\,dt$ $\int_0^\pi \cos(t) + \sin(t)\,dt$ $\int_0^\pi \sin^2(t) - \cos^2(t)\,dt$ $\int_0^\pi dt$

Compute the line integral $\int_y \,ds$ where is the curve parameterized by $(t) = t^4 \subset -t^2 \supset$, $0 \le t \le 1$.

$\frac{1}{12}(5^{3/2}-1)$ $\frac{1}{6}(3^{1/2}-1)$ $\frac{1}{3}(2^{3/2}-1)$ $\frac{1}{4}(3^{5/2}-1)$ $\frac{1}{2}(6^{1/2}-1)$

Let be the curve parameterized by $(t) = \cos(3t) \subset +\sin(2t) \supset +t$, $0 \le t \le 2\pi$. Calculate $\int_y e^{xy}\,dx + xe^{xy}\,dy + \,dz$.

2π 1 0 $e - 2\pi$ e

Determine which of the following integrals gives the area of the surface $z^4 = 4(x^2+y^2)$, $0 \le z \le \sqrt{2}$.

$\int_0^{2\pi}\int_0^1 \sqrt{r^2 + r/2}\, dr\, d\theta$ $\int_0^{2\pi}\int_0^1 4r\sqrt{4r^2 + z^2}\, dr\, d\theta$ $\int_0^{2\pi}\int_0^1 8\sqrt{2}r\, dr\, d\theta$ $\int_0^{2\pi}\int_0^1 4r^2\sqrt{2}\, dr\, d\theta$ $\int_0^{2\pi}\int_0^1 4r^2\sqrt{4r^2 + r}\, dr\, d\theta$

Let Σ be the portion of the paraboloid $z = x^2 + y^2$ in the first octant below $z = 2$ and let be the unit downward normal vector to Σ. Compute the flux integral

$$_{\Sigma}(y \subset -x \supset +) \cdot \, d\sigma$$

$-\frac{\pi}{2}$ $\frac{4\pi}{3}$ 0 4π $-\frac{5\pi}{2}$

Let Σ be the portion of the parabolic sheet $z = 1 - x^2$ that lies inside the square $-1 \leq x \leq 1$, $-1 \leq y \leq 1$, and let be the boundary of Σ oriented counterclockwise when viewed from above. Use Stokes' Theorem to rewrite the line integral $\int_{(} y \subset +z \supset +x) \cdot d$ as a double integral.

$-\int_{-1}^{1}\int_{-1}^{1}(2x+1)\,dy\,dx$ $\int_{-1}^{1}\int_{-1}^{1}(\subset + \supset +)\,dy\,dx$ $\int_{-1}^{1}\int_{-1}^{1}(x+y+x^2)\,dy\,dx$ $-\int_{-1}^{1}\int_{-1}^{1}(2xy+x)\,dy\,dx$ $\int_{-1}^{1}\int_{-1}^{1}(y \subset +z \supset +x)\sqrt{4x^2+1}\,dy\,dx$

Let D be the solid cylinder defined by $x^2 + y^2 \leq 4$, $0 \leq z \leq 3$. Let Σ be the boundary of this solid and let be its outward unit normal vector. Use the Divergence Theorem to compute the flux integral $_{\Sigma}(xy \subset -y^2 \supset +z^2) \cdot d\sigma$.

36π 18π 12π 9π 6π