MATH 225: Calculus III Name:____________________

Exam I *September 25, 2001* Instructor:____________________

Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this page. There are 8 multiple choice questions worth 5 points each and 4 partial credits problems worth 10 points each. You start with 20 points. On the partial credit problems *you must show your work and all important steps to receive credit.*

You may use a calculator if you wish.

Find the angle between the planes $x - y + 2z = 3$ and $2x + y + z = -5$.

$\pi/3$ $\pi/6$ $\pi/2$ $3\pi/4$ $\pi/4$

Compute the arc length of the curve parameterized by $\mathbf{r}(t) = 2t\mathbf{i} + t^2\mathbf{j} + \frac{1}{3}t^3\mathbf{k}$ for $0 \leq t \leq 1$.

$\frac{7}{3}$ $\frac{\sqrt{46}}{3}$ $\frac{17}{12}$ $\frac{10}{3}$ $\frac{\sqrt{161}}{12}$

Find the parametric equations of the line tangent to the curve $\mathbf{r}(t) = \langle t^2, t, \ln t\rangle$ at the point $(1, 1, 0)$.

$x = 1+2t$, $y = 1+t$, $z = t$ $x = 2t$, $y = 1$, $z = \frac{1}{t}$ $x = 2+t$, $y = 1+t$, $z = 1$ $x = 1+2t^2$, $y = 1+t$, $z = 1$ $x = 1+t$, $y = 1+2t$, $z = -t$

The position of a moving particle at time t is given by $\mathbf{r}(t) = \langle \cos(2t), t, \sin(2t)\rangle$. Determine the tangential and normal components of the acceleration at $t = 0$.

$a_T = 0$, $a_N = 4$ $a_T = -4$, $a_N = 0$ $a_T = \sqrt{5}$, $a_N = \sqrt{11}$ $a_T = \sqrt{5}$, $a_N = 0$ $a_T = 1$, $a_N = \sqrt{5}$

Determine which of the following plots represents the curve traced by the vector function $\mathbf{r}(t) = \langle 2\cos(4t), 2\sin(4t), t\rangle$.

Determine which of the following symmetric equations gives a line that is parallel to the vector $\mathbf{i} + 2\mathbf{j} + (1/2)\mathbf{k}$.

$2(x+1) = y - 3 = 4(z-1)$ $x + 1 = 2y + 1 = (1/2)z + 1$ $2x = y = (1/2)z$ $x = 2y$ and $z = 1$ $y = (1/4)z$ and $x = 0$

Find a vector that is orthogonal to the plane containing the points $(1, 0, 1)$, $(2, -1, 1)$, and $(0, 0, 3)$.

$2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ $-\mathbf{i} + \mathbf{k}$ $2\mathbf{i} - \mathbf{j} - 2\mathbf{k}$ $2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$ $\mathbf{i} + \mathbf{j} - \mathbf{k}$

Determine the function that corresponds to the following contour map.

$f(x,y) = ye^x$ $f(x,y) = xy - 1$ $f(x,y) = x^2 - y$ $f(x,y) = ye^{-x}$ $f(x,y) = x - y^2$

9. Find a vector function that traces out the curve of intersection of the ellipsoid $(x-1)^2 + 3(y-1)^2 + (z-1)^2 = 1$ and the plane $y = x$.

10. The force acting on a moving particle of mass $m = 2$ at time t is

$$\mathbf{F}(t) = e^t\mathbf{i} + e^{-t}\mathbf{j} + t\mathbf{k}$$

If the particle's initial position and velocity are $\mathbf{r}(0) = \langle 0, 0, 1\rangle$ and $\mathbf{v}(0) = \langle 0, 1, 0\rangle$, respectively, determine the particle's position vector, $\mathbf{r}(t)$, for any $t \geq 0$.

11. Let $\mathbf{r}(t) = t\mathbf{i} + t\mathbf{j} + \frac{t^2}{2}\mathbf{k}$. Find the unit tangent vector, $\mathbf{T}(0)$, the unit normal vector, $\mathbf{N}(0)$, and the unit binormal vector $\mathbf{B}(0)$ at $t = 0$.

12. Evaluate the limit

$$\lim_{(x,y)\to(0,0)} \frac{y^4 + 2x^3y}{(x^2 + y^2)^2}$$

or show it does not exist.