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MATH 225: Calculus III Name:____________________

Exam II *October 30, 2001* Instructor:____________________

Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this page. There are 8 multiple choice questions worth 5 points each and 4 partial credits problems worth 10 points each. You start with 20 points. On the partial credit problems *you must show your work and all important steps to receive credit.*

You may use a calculator if you wish.

Find the directional derivative of $f(x,y) = e^{x\sin(y)}$ at $P(1,0)$ in the direction of $Q(-2,4)$.

$4/5$ $-3/5$ -2 0 4

Suppose the plane $2x - y + z = 0$ is tangent to the level surface $f(x,y,z) = 2$ at the point $(0,1,1)$. Determine which of the vectors below is parallel to $\nabla f(0,1,1)$.

$\langle -2,1,-1\rangle$ $\langle 1,2,-1\rangle$ $\langle 2,-1,-1\rangle$ $\langle 4,-2,-1\rangle$ $\langle 4,2,-2\rangle$

Let $f(x,y) = 8x^3 + 6xy + y^3$. Determine which of the following statements is true.

$(0,0)$ is a saddle point and $(-1/2,-1)$ is a local maximum. $(0,0)$ is a saddle point and $(-1/2,-1)$ is a local minimum. $(0,0)$ is a local maximum and $(-1/2,-1)$ is a local minimum. $(0,0)$ is a local minimum and $(-1/2,-1)$ is a local maximum. $(0,0)$ is a local maximum and $(-1/2,-1)$ is a saddle point.

Let C be the curve of intersection of the surfaces $x^2 + y^2 + z^2 = 1$ and $xy + z^2 = 0$. Determine which of the following systems of equations must be solved to find the point(s) on C closest to $(1,1,1)$ using Lagrange multipliers.

$$\begin{aligned}
2(x-1) &= 2\lambda x + \mu y\\
2(y-1) &= 2\lambda y + \mu x\\
(z-1) &= \lambda z + \mu z\\
x^2 + y^2 + z^2 &= 1\\
xy + z^2 &= 0
\end{aligned}$$

$$\begin{aligned}
2x &= 2\lambda(x-1) + \mu y\\
2y &= 2\lambda(y-1) + \mu x\\
2z &= \lambda(z-1) + \mu z\\
x^2 + y^2 + z^2 &= 1\\
xy + z^2 &= 0
\end{aligned}$$

$$(x-1) = \lambda x$$
$$(y-1) = \lambda y$$
$$(z-1) = \lambda z$$
$$x^2 + y^2 - xy = 1$$

$$2(x-1) = \lambda y$$
$$2(y-1) = \lambda x$$
$$(z-1) = \lambda z$$
$$x^2 + y^2 - xy = 1$$

$$2(x-1) = \lambda(2x-y)$$
$$2(y-1) = \lambda(2y-x)$$
$$2(z-1) = \lambda(x+y)$$
$$x^2 + y^2 - xy = 1$$

Reverse the order of integration in the double integral $\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} f(x,y)\ dy\ dx$.
$\int_0^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} f(x,y)\,dx\,dy$ $\int_0^{\sqrt{1-x^2}} \int_{-1}^{1} f(x,y)\,dx\,dy$ $\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_0^1 f(x,y)\,dx\,dy$ $\int_{-1}^{1} \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} f(x,y)\,dx$ d
$\int_{-1}^{1}\int_0^{\sqrt{1-y^2}} f(x,y)\ dx\ dy$

Evaluate the double integral $\int_0^1 \int_0^{x^2} xe^y\ dy\ dx$.
$\frac{1}{2}e-1$ $\frac{1}{2}(e-1)$ $\frac{1}{2}e$ $e-\frac{1}{2}$ $e-1$
Find the area inside the curve $r = 2+\cos(\theta)$. $\frac{9\pi}{2}$ $\frac{3\pi}{2}$ 6π $\frac{15\pi}{2}$ 3π
Determine which of the following double integrals gives the area of the part of the surface $z = x + y^3$ that lies above the region bounded by $x = y^2$, $y = 0$, and $x = 1$.
$\int_0^1 \int_{y^2}^1 \sqrt{2+9y^4}\,dx\,dy$ $\int_0^1 \int_0^{\sqrt{x}} \sqrt{2+3y^2}\,dy\,dx$ $\int_0^1 \int_{\sqrt{x}}^1 \sqrt{2+3y^2}\,dy\,dx$ $\int_0^1 \int_0^{y^2} \sqrt{2+9y^4}\,dx\,dy$
$\int_0^1 \int_{y^2}^1 \sqrt{1+x^2+9y^4}\,dx\,dy$

9. A triangular sheet of glass is expanding. When the base is 2 in and the height is 4 in, the base is increasing at the rate of 0.25 in/hr and the height at 0.5 in/hr. At what rate is the area of the triangle increasing?

10. Find the absolute maximum and minimum values of $f(x, y) = x^2 + y^2 + x + 1$ on the unit disk $x^2 + y^2 \leq 1$.

11. Let D be the triangular region in the plane with vertices $(0,0)$, $(2,0)$, and $(1,2)$. Compute $\iint_D x dA$.

12. A lamina of uniform density occupies the region D bounded by the parabola $y = 1 - x^2$ and the x-axis. Find its center of mass.