MATH 225: Calculus III Name:________________________________

Exam III *November 29, 2001* Instructor:________________________________

Record your answers to the multiple choice problems by placing an $\times$ through one letter for each problem on this page. There are 8 multiple choice questions worth 5 points each and 4 partial credits problems worth 10 points each. You start with 20 points. On the partial credit problems *you must show your work and all important steps to receive credit.*

You may use a calculator if you wish.

Let E be the tetrahedron with vertices $(0,0,0)$, $(1,0,0)$, $(0,2,0)$, and $(0,0,2)$. Express the integral $\iiint_E f(x,y,z)\,dV$ as an iterated integral.

$\int_0^1 \int_0^{2-2x} \int_0^{2-2x-y} f(x,y,z)\,dz\,dy\,dx$ $\int_0^1 \int_0^{1-y} \int_0^{(2-y-z)/2} f(x,y,z)\,dx\,dz\,dy$ $\int_0^1 \int_0^2 \int_0^2 f(x,y,z)\,dz\,dy\,dx$ $\int_0^2 \int_0^{1-y} \int_0^{2-x-2y} f(x,y,z)\,dz\,dx\,dy$ $\int_0^2 \int_0^1 \int_0^{2-y} f(x,y,z)\,dz\,dx\,dy$

Let E be the region between the spheres $x^2+y^2+z^2 = z$ and $x^2+y^2+z^2 = 2z$. Which of the following represents $\iiint_E (x^2+y^2)\;dV$ in spherical coordinates.

$\int_0^{2\pi} \int_0^{\pi/2} \int_{\cos(\phi)}^{2\cos(\phi)} \rho^4 \sin^3(\phi)\,d\rho\,d\phi\,d\theta$ $\int_0^{2\pi} \int_0^{\pi/2} \int_{\cos(\phi)}^{2\cos(\phi)} \rho^2 \sin(\phi)\,d\rho\,d\phi\,d\theta$ $\int_0^{2\pi} \int_0^{\pi/2} \int_1^2 \rho^4 \sin(\phi)\,d\rho\,d\phi\,d\theta$ $\int_0^{2\pi} \int_0^{\pi/2} \int_{\cos(\theta)}^{2\cos(\theta)} \rho^4 \sin^3(\theta)\,d\rho\,d\phi\,d\theta$ $\int_0^{2\pi} \int_{-\pi/2}^{\pi/2} \int_{\cos(\phi)}^{2\cos(\phi)} \rho^4 \sin^3(\phi)\,d\rho\,d\phi\,d\theta$

Let $\mathcal{C}$ be the curve parameterized by $\mathbf{r}(t) = \langle t, t^2, t^2 \rangle$, $0 \le t \le 1$. Evaluate the line integral $\int_{\mathcal{C}} x\,ds$.

$13/12$ $91/3$ 0 $1/2$ $(5^{3/2}-1)/24$

Let $\mathbf{F}(x,y) = (x+y)\mathbf{i} + (y-x)\mathbf{j}$. Calculate $\int_{\mathcal{C}} \mathbf{F}\cdot d\mathbf{r}$ where $\mathcal{C}$ is the circle $\mathbf{r}(t) = \langle \cos(t), \sin(t) \rangle$, $0 \le t \le 2\pi$.

-2π 0 $3\pi/2$ -1 π

Let $\mathbf{F}$ be a vector field defined on $\mathbf{R}^3$. Determine which of the following conditions guarantees that $\mathbf{F} = \nabla f$ for some function f.

$\operatorname{curl}\mathbf{F} = \mathbf{0}$ $\operatorname{div}\mathbf{F} = 0$ $\operatorname{div}\operatorname{curl}\mathbf{F} = 0$ $\operatorname{curl}\operatorname{curl}\mathbf{F} = \mathbf{0}$ $\operatorname{grad}\operatorname{div}\mathbf{F} = \mathbf{0}$

Let $\mathbf{F} = (e^x + yz)\mathbf{i} + (e^y - xz)\mathbf{j} + (\sin(z) + x - y)\mathbf{k}$. Compute $\operatorname{div}\mathbf{F}$.

$e^x + e^y + \cos(z)$ $e^x\mathbf{i} + e^y\mathbf{j} + \cos z\mathbf{k}$ $z - x + 1$ $y + z + \cos z$ $(x-1)\mathbf{i} + (y-1)\mathbf{j} + 2z\mathbf{k}$

Determine which of the following integrals gives the area of the surface parameterized by $\mathbf{r}(u,v) = u^2\mathbf{i} + uv\mathbf{j} + v^2\mathbf{k}$, $0 \le u \le 1$, $0 \le v \le 2$.

$\int_0^2 \int_0^1 2\sqrt{u^4 + 4u^2v^2 + v^4}\,du\,dv$ $\int_0^2 \int_0^1 \sqrt{u^4 + u^2v^2 + v^4}\,du\,dv$ $\int_0^2 \int_0^1 u^4 + u^2v^2 + v^4\,du\,dv$ $\int_0^2 \int_0^1 \sqrt{2}(u^2 + 4uv + v^2)\,du\,dv$ $\int_0^2 \int_0^1 2(u^2+v^2)\,du\,dv$

Determine which of the following plots is the surface parameterized by

$$\mathbf{r}(u,v) = u\cos(v)\mathbf{i} + u\sin(v)\mathbf{j} + v\mathbf{k}, \quad -1 \le u \le 1, 0 \le v \le 2\pi$$

9. Let E be the region between the paraboloids $z = x^2 + y^2$ and $z = 2 - x^2 - y^2$. Compute $\iiint_E \sqrt{x^2 + y^2}\, dV$.

10. Let $\mathbf{F} = (e^y + yz)\mathbf{i} + x(e^y + z)\mathbf{j} + (xy - 2)\mathbf{k}$.

a) Find a function f such that $\mathbf{F} = \nabla f$.

b) Use the Fundamental Theorem for Line Integrals to evaluate

$$\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$$

where $\mathcal{C}$ is a curve from $(0, 0, 0)$ to $(3, 1, -1)$.

11. Let R be the region bounded by the hyperbolas $xy = 1$ and $xy = 9$ and the lines $y = x$ and $y = 4x$ in the first quadrant. Use the change of variables $x = uv$, $y = u/v$, $u \geq 0$, $v \geq 0$, to evaluate the integral $\iint_R x^2 y \, dA$.

12. Use Green's Theorem to calculate

$$\int_C (e^{x^2} + xy)dx + (x + \sin(y^2))dy$$

where C is the square with vertices $(0,0)$, $(1,0)$, $(1,1)$, $(0,1)$, oriented counter-clockwise.